NONLINEAR ACTIVE NOISE CONTROL USING VOLTERRA FILTERING WITH A VARIABLE STEP-SIZE GS-PAP ALGORITHM

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ABSTRACT

In this paper, a nonlinear active noise control (ANC) is proposed using adaptive Volterra filtering with a variable step-size Gauss-Seidel pseudo affine projection (VSSGS-PAP) algorithm. In particular, it requires a lower computational burden than the VSS affine projection (VSSAP) algorithm, and provides an improved convergence as the VSSAP algorithm does. Simulation results show that the proposed approach provides lower misalignment performance than conventional linear approaches.

Index Terms— Active noise control (ANC), Gauss-Seidel pseudo affine (GS-PAP) algorithm, adaptive Volterra filtering, variable step-size

1. INTRODUCTION

Recently, active noise control (ANC) systems have been increasingly developed [1-2]. A common single-channel active noise controlling scheme is depicted in Fig. 1. To reduce computational cost, many adaptive algorithms have been employed for the ANC systems. Among them, the affine projection (AP) algorithm has been suggested as a relation between the least mean square (LMS) algorithm and the recursive least squares (RLS) algorithm [3-5]. The AP algorithm provides faster convergence and higher stability than LMS and requires much fewer computations than the RLS method. Accordingly, the AP algorithm has been widely used for ANC systems [1-2]. However, if the affine projection order is increased, then the total computational complexity can be more increased or uncontrolled [3]. To solve the computational complexity and stability problems, the GS-PAP algorithm is proposed [3,5,9]. However, the performance of the GS-PAP algorithm can be poor in the case of a step size with a non-unity variance being chosen [9]. Recently, a VSSGS-PAP algorithm with better convergence was proposed to solve the problem [5].

In practice, since a nonlinear ANC system is called for in case of a nonlinear primary path, conventional linear adaptive filtering approaches should be extended for nonlinear compensation [6]. To solve this problem, adaptive Volterra filtering approaches were proposed [7-8], whereby a Volterra system has a linear relation between Volterra kernels and the system output. This observation makes it possible to extend conventional linear adaptive filtering methods further for nonlinear Volterra ANC systems.

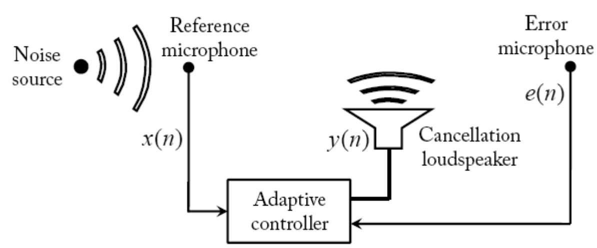

Fig. 1. Active noise control

In this paper, a nonlinear active noise control (ANC) is proposed using adaptive Volterra filtering with a variable step-size Gauss-Seidel pseudo affine projection (VSSGS-PAP) algorithm, yielding lower misalignment performance than conventional linear approaches.

This paper is organized as follows: In section 2, explain the Volterra ANC method. In section 3, a nonlinear active noise control (ANC) is proposed by employing Volterra filtering with a VSSGS-PAP algorithm. Also, simulation results are provided in section 4, and finally, section 5 concludes this work.

2. A VOLTERRA NONLINEAR ANC SYSTEM

The input-output relation of a third-order Volterra model can be expressed by [6]

$$\begin{aligned} y[n] &= \sum_{m_1=0}^{N-1} w_1[m_1]x[n-m_1] + \sum_{m_1=0}^{N-1}\sum_{m_2=0}^{N-1} w_2[m_1,m_2]x[n-m_1]x[n-m_2] \\ &+ \sum_{m_1=0}^{N-1}\sum_{m_2=m_1}^{N-1}\sum_{m_3=m_2}^{N-1} w_3[m_1,m_2,m_3]x[n-m_1]x[n-m_2]x[n-m_3] \quad (1) \end{aligned}$$

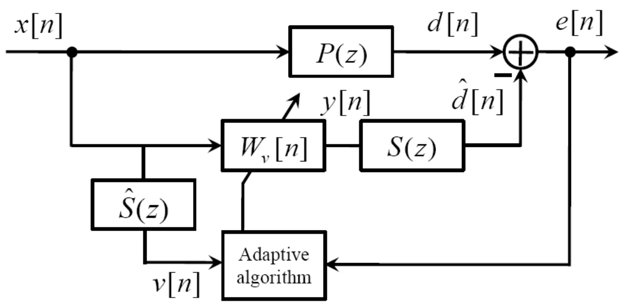

Fig.2. The proposed filtered-x active noise controller

Since the output of the Volterra filter is linear with respect to Volterra kernels, the existing linear theory can be directly applied to Volterra filtering for nonlinear ANC systems [6-7]. In particular, a block diagram for a filtered-x Volterra ANC system is shown in Fig. 2, where $\mathbf{w}_v$ is the Volterra kernel vector and $x[n]$ is an input received by a reference microphone as shown in Fig. 1. Also, $P(z)$ corresponds to a nonlinear primary path, and $x[n]$ passes through the Volterra filter $\mathbf{w}_v$ and the secondary path $S(z)$. By adaptively minimizing the difference between the desired signal $d[n]$ and the estimated signal $\hat{d}[n]$ from the adaptive Volterra filter, we can obtain a residual signal $e[n]$ received by another error microphone as shown in Fig. 1. Furthermore, the adaptive Volterra filter coefficients can be updated by utilizing the filtered input signal $v[n]$ (estimated by a secondary path $\hat{S}(z)$) and the error signal $e[n]$ as shown in Fig. 2. Accordingly, the Volterra filtering with a filtered-x ANC method can be expressed as follows:

$$\mathbf{x}_v[n] = [\,x[n],\ldots,x[n-N+1],\;x^2[n],x[n]x[n-1],\ldots,x[n]x[n-N+1],\; x^2[n-1],\ldots,x[n-1]x[n-N+1],x[n-2]x[n-2],\ldots,x^2[n-N+1],\; x^3[n],\ldots,x^2[n]x[n-N+1],\ldots,x^3[n-N+1]\,]^T \quad (2)$$

$$\mathbf{w}_v[n] = [\;w_1[0],\ldots,w_1[N-1],w_2[0,0],\ldots,w_2[0,N-1],\;w_2[1,1],\ldots,w_2[1,N-1],\; w_2[2,2],\ldots,w_2[N-1,N-1],w_3[0,0,0],\ldots,w_3[0,0,N-1],\ldots,\; w_3[0,0,N-1],\ldots,w_3[N-1,N-1,N-1]\;]^T \quad (3)$$

where $\mathbf{x}_v[n]$ is a Volterra input, and $\mathbf{w}_v[n]$ is a Volterra kernel vector. As shown in Fig. 2, a Volterra filter output $y[n]$, a desired signal $d[n]$ (i.e., the output of a nonlinear primary path), and an error signal $e[n]$ can be written as

$$y[n] = \mathbf{w}_v^T[n]\,\mathbf{x}_v[n] \quad (4)$$

$$\hat{d}[n] = S * y[n] \quad (5)$$

$$e[n] = d[n] - \hat{d}[n] \quad (6)$$

In addition, an update equation for the Volterra filter coefficients in the FXLMS algorithm is expressed as [7]

$$\mathbf{w}_v[n+1] = \mathbf{w}_v[n] + \mu\,\mathbf{v}_v[n]e[n] \quad (7)$$

$$\mathbf{v}_v[n] = [\,v[n],v[n-1]\ldots v[n-N+1]\;v^2[n]\;v[n]v[n-1]\ldots v[n]v[n-L+1]\; v^2[n-1],\ldots,v[n-1]v[m-N+1],v[n-2]v[n-2],\ldots,v^2[n-N+1]\; v^3[n],\ldots,v^2[n]v[n-N+1],\ldots,v^3[n-N+1]]^T \quad (8)$$

$$v[n] = \hat{S} * x[n] \quad (9)$$

where μ ($0<\mu<1$) is a step-size controlling the convergence speed of adaptive filter. In (9), $v[n]$ is a filtered signal from the estimated secondary path $\hat{S}$, and is formulated for Volterra input vector $\mathbf{v}_v[n]$ as in (8).

3. NONLINEAR ANC WITH A VSSGS-PAP ALGORITHM

In this section, a nonlinear Volterra ANC with a VSSGS-PAP algorithm is considered. More specifically, the VSSGS-PAP algorithm solves the step-size problem of the conventional GS-PAP algorithm using the filtered error signal by linear prediction coefficients [5,9]. Furthermore, the VSSGS-PAP algorithm yields faster convergence and lower misalignment by utilizing the variable step-size [4] than the conventional GS-PAP algorithm. However, in practical ANC systems, nonlinearities can occur in the primary and/or secondary paths. Also, linear adaptive filter approaches have some limitation in compensating for the nonlinearity of the corresponding nonlinear path. Therefore, a Volterra filtering with a VSSGS-PAP algorithm is proposed in this paper by applying the linear filter theory to nonlinear ANC systems, because the Volterra filter has the linear relationship between the Volterra filter output and the Volterra kernel vector. That is, the Volterra filtering derived by using a VSSGS-PAP algorithm in [5] can be expressed as

$$R_v[n] = X_v^T[n]X_v[n] + \delta\mathbf{I} \quad (10)$$

$$X_v[n] = [\,\mathbf{x}_v[n]\;\mathbf{x}_v[n-1]\ldots\mathbf{x}_v[n-M+1]] \quad (11)$$

$$R_v[n]\mathbf{p}[n] = \mathbf{b} \quad (12)$$

$$\mathbf{w}_v[n+1] = \mathbf{w}_v[n] + \mu_{PAP}[n]\frac{\varepsilon_v[n]}{\|\mathbf{u}_v[n]\|_2+\delta}\mathbf{u}_v[n] \quad (13)$$

$$\mathbf{u}_v[n] = \frac{1}{P_0}\sum_{i=0}^{M-1}P_i[n]\mathbf{x}_v[n-i] \quad (14)$$

$$\varepsilon_v[n] = \frac{1}{P_0}P[n]\mathbf{e}_v[n] \quad (15)$$

$$\mathbf{e}_v[n] = [\ e(n)\ e(n-1)\ \ldots e(n-M+1)] \quad (16)$$

$$\mu_{PAP}[n] = \mu_{\max} \frac{\sigma_\varepsilon^2[n]}{\sigma_\varepsilon^2[n] + C} \quad (17)$$

where $R_v[n]$ is an input autocorrelation matrix and δ is a regularization factor to avoid the autocorrelation matrix becoming ill-conditioned. Also, $\mathbf{I}$ is an identity matrix and M is an AP order. In (12), the linear prediction problem can be solved by using Gauss-Seidel iteration, and $\mathbf{p}[n]$ corresponds to the optimal linear prediction coefficients. Furthermore, $\mathbf{b}$ is a vector with unit value only at the first element and other elements being zero. In (14), $\mathbf{u}_v[n]$ is an approximated decorrelation vector to update the GS-PAP algorithm, and in (15), $\varepsilon_v[n]$ is a filtered error vector. In (17), $\mu_{PAP}[n]$ is a variable step-size value [3, 4], $\sigma_\varepsilon^2[n]$ is a squared expectation of filtered error signal vector $\mathrm{E}\{\varepsilon_v^2[n]\}$, and C is a constant value. When $\sigma_\varepsilon^2[n]$ is large compared to the C, $\mu_{PAP}[n]$ is closed to $\mu_{\max}$. But as $\sigma_\varepsilon^2[n]$ becomes smaller, so does $\mu_{PAP}[n]$ [5].

4. SIMULATION RESULTS

In this section, we demonstrate the performance of the proposed approach by using a simulation. Here, a nonlinear primary path is assumed to be the following third-order polynomial model [7].

$$d[n] = t[n] + 0.08 \cdot t^2[n] - 0.04 \cdot t^3[n] \quad (18)$$

$$t(n) = \mathbf{f}^T \mathbf{x}[n] \quad (19)$$

where $\mathbf{f}$ is an impulse response vector of the primary path. An input signal $x[n]$ is chosen to be a white Gaussian noise which has zero mean and unit variance. Also, an AP order and a maximum step-size $\mu_{\max}$ are set to 5 and 1, respectively. For quantitative performance evaluation, the normalized mean square error (NMSE) vs. number of iterations, calculated by each adaptive filter algorithm, is utilized, given by

$$\mathrm{NMSE} = 10 \log_{10} \frac{E\{e^2[n]\}}{\sigma_d^2} \quad (20)$$

In (20), $E\{e^2[n]\}$ and σ_d^2 denote the power of an error signal and the power of a noise signal passed through the primary path, respectively.

Fig. 3 shows the NMSE curves in case of a *linear* VSSGS-PAP algorithm and the proposed *third-order Volterra* VSSGS-PAP algorithm, where the convergence level by the linear VSSGS-PAP algorithm is -20 dB, but that by the proposed Volterra VSSGS-PAP algorithm -40 dB. Therefore, it demonstrates that the proposed approach leads to better ANC performance than the linear VSSGS-PAP algorithm for the nonlinear ANC.

5. CONCLUSION

In this paper, a nonlinear ANC method is proposed by employing Volterra filtering with a VSSGS-PAP algorithm. It is demonstrated that the proposed approach provides a better ANC performance for the nonlinear ANC system.

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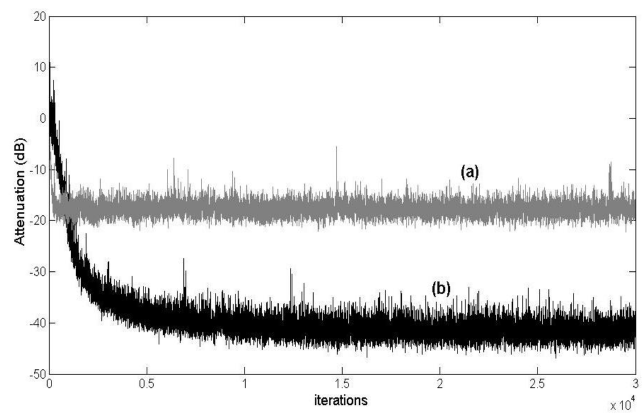

Fig. 3. NMSE curves in case of (a) linear VSSGS-PAP algorithm and (b) Volterra VSSGS-PAP algorithm

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