INSTANTANEOUS BLIND SIGNAL EXTRACTION USING SECOND ORDER STATISTICS

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ABSTRACT

The ultimate goal of instantaneous blind signal extraction is to find one source out of an instantaneous mixture of many others, without, or with a minimum of, prior information. Extraction can be performed by first identifying the complete mixing system and subsequently inverting that system. The goal of this paper is to describe the problems behind blind extraction and to directly find the extracting solution, without first identifying the complete mixing system. The proposed method uses second order statistics to identify the extracting solution and can be applied to mixing problems with different kinds of temporal structure e.g. non-stationarity, coloredness.

Index Terms— Blind Signal Processing, Blind Signal Extraction, Second Order Statistics.

1. INTRODUCTION

In the field of 'Blind Signal Processing' several subproblems, e.g. Blind Source Separation (BSS), Blind Identification (BI), and Blind Signal Extraction (BSE), can be identified. These individual problems attained a lot of attention, for example in [1–5] and the references therein. The given problems apply to several kinds of mixing problems e.g. instantaneous and convolutive mixtures, which are over-determined, under-determined, or square. The procedures to solve the individual problems rely on various assumptions like stationarity or sparseness and use different properties like second order statistics, higher order statistics, etc. This work uses the framework described in [1, 2]. It has been proven that the framework works for BI and we show that, with small adjustments, we can find extraction vectors of a square or over-determined mixing system, without first identifying the complete mixing system. Well known mathematical tools like SVD and eigenvalue decomposition are used and at this moment the proposed method finds the BSS solution. When the method is implemented for batch, online or realtime operation, the algorithm should be adjusted to only and directly select the desired solution. The given framework allows for these adjustments.

This paper is organized as follows. First the model is described and all assumptions are given in Section 2. Subsequently the ideal solution to extract the desired source is given in Section 3. A method to find the extracting solution is described in Section 4. In Section 5 simulation results are given and finally conclusions and future research are discussed in Section 6.

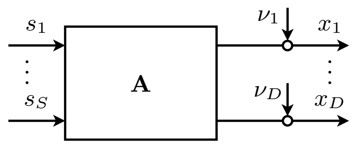

Fig. 1: Instantaneous mixing model

2. MIXING MODEL AND ASSUMPTIONS

A graphical representation of the instantaneous mixing system is given in Fig. 1. The system can be described by a real-valued mixing matrix $\mathbf{A}$, and therefore we can give a mathematical representation of the mixing system by:

$$\mathbf{x}[n] = \sum_{j=1}^{S} \mathbf{a}^j s_j[n] + \boldsymbol{\nu}[n] = \mathbf{A}\mathbf{s}[n] + \boldsymbol{\nu}[n] \quad \forall\, n \in \mathbb{Z}, \tag{2.1}$$

where:

$$\mathbf{x}[n] \triangleq \begin{bmatrix} x_1[n] \\ \vdots \\ x_D[n] \end{bmatrix}, \mathbf{s}[n] \triangleq \begin{bmatrix} s_1[n] \\ \vdots \\ s_S[n] \end{bmatrix}, \boldsymbol{\nu}[n] \triangleq \begin{bmatrix} \nu_1[n] \\ \vdots \\ \nu_D[n] \end{bmatrix}, \mathbf{a}^j \triangleq \begin{bmatrix} a_1^j \\ \vdots \\ a_D^j \end{bmatrix}$$

are real-valued column vectors representing the D sensor signals, S source signals, D noise signals, and S independent mixing columns of length D, respectively. Putting all mixing columns together in a matrix results in the $D \times S$ mixing matrix given by $\mathbf{A} = \begin{bmatrix} \mathbf{a}^1 & \cdots & \mathbf{a}^S \end{bmatrix}$.

Notation, as already used in the equations above, of matrices and vectors is in bold upper and lower case letters, respectively. Subscript and superscript indices refer to row and column elements in a vector, respectively. Elements of a matrix, in lower case letters, are indexed with both subscript and superscript for the row and column index, respectively. Using this notation, one element of (2.1) can therefore be written as:

$$x_i[n] = \sum_{j=1}^{S} a_i^j s_j[n] + \nu_i[n] \quad \forall\, n \in \mathbb{Z}, \quad \forall\, 1 \leq i \leq D.$$

Furthermore, the discrete time index $n \in \mathbb{Z}$ denotes the sample number.

Our method exploits Second Order Temporal Structure (SOTS) of the data. Therefore, we have to make a number of assumptions on the SOS of the source and noise signals. First of all we define the operators which are used to acquire the SOS.

Definition 2.1. *The correlation function of source signal pair* (s_{i_1}, s_{i_2}) $\forall 1 \leq i_1, i_2 \leq S$ *at a given time* n *and with a certain lag* k *is given by:*

$$r^s_{i_1 i_2}[n,k] \triangleq \mathrm{E}\left\{s_{i_1}[n]s_{i_2}[n-k]\right\} \quad \forall n,k \in \mathbb{Z},$$

with $\mathrm{E}\{\cdot\}$ *the mathematical expectation operator.*

The correlation functions can be approximated by averaging over a block of stationary data. If non-stationary sources are present, we can use multiple blocks, in which the sources are temporally approximately stationair, to estimate the statistics. The time index n therefore represents the statistics in the n'th block of data.

In a similar way as the source signal correlation functions, the noise and sensor correlation functions and source-noise cross-correlation functions for time-lag pair $[n,k]$ are defined by:

$$\begin{aligned} r^{\nu}_{i_1 i_2}[n,k] &\triangleq \mathrm{E}\left\{\nu_{i_1}[n]\nu_{i_2}[n-k]\right\}; \\ r^{x}_{i_1 i_2}[n,k] &\triangleq \mathrm{E}\left\{x_{i_1}[n]x_{i_2}[n-k]\right\} \\ r^{s\nu}_{ij}[n,k] &\triangleq \mathrm{E}\left\{s_i[n]\nu_j[n-k]\right\}. \end{aligned}$$

Using these definitions, we are able to define a so-called Noise-Free Region of Support (Ω):

Definition 2.2. *The Noise-Free Region of Support (ROS), also denoted by* Ω*, is a set of time-lag pairs* (n,k) *for which the noise correlation functions and the source and source-noise cross-correlation functions, at these pairs, equal zero:*

$$\forall (n,k) \in \Omega : \begin{cases} r^s_{i_1 i_2}[n,k] = 0 & \forall 1 \leq i_1 \neq i_2 \leq S; \\ r^{\nu}_{i_1 i_2}[n,k] = 0 & \forall 1 \leq i_1, i_2 \leq D; \\ r^{s\nu}_{ij}[n,k] = 0 & \forall 1 \leq i \leq S,\ 1 \leq j \leq D. \end{cases}$$

From these assumptions it follows that in theory the noise signals do not influence the sensor correlation functions, when we take the time-lag pairs from the ROS.

Since we need the source correlation matrix to be full rank, we have to make an additional assumption. The source auto-correlation functions have to be linearly independent in Ω:

$$\sum_{i=1}^{S} \xi^i r^s_{ii}[n,k] = 0 \quad \Longleftrightarrow \quad \xi^i = 0 \quad \forall 1 \leq i \leq S. \tag{2.2}$$

3. SOLUTION OF THE EXTRACTION PROBLEM

Using the model described in (2.1), the goal of blind extraction is to find one desired source out of a square or over-determined mixture of many others, by identifying a row vector $\mathbf{w}_i$ such that:

$$\hat{s}_i[n] = \mathbf{w}_i\mathbf{x}[n] = \mathbf{w}_i\mathbf{A}\mathbf{s}[n] + \mathbf{w}_i\boldsymbol{\nu}[n],$$

where $\hat{s}_i[n]$ is the estimation of the desired source. Since blind processing can be performed up to the well known scaling and permutation indeterminacy, we can say without loss of generality, that we want to find the first source signal, i.e. find a vector $\mathbf{w}_1$ such that:

$$\hat{s}_1[n] = \mathbf{w}_1\mathbf{x}[n] = \mathbf{w}_1\mathbf{A}\mathbf{s}[n] + \mathbf{w}_1\boldsymbol{\nu}[n]. \tag{3.1}$$

If the estimation is performed completely blind, we cannot identify whether the desired source is extracted. If some a priori knowledge is present, we can try to extract only the desired source, but the focus in this paper is on developing a method which can be modified by enclosing a priori knowledge to select the desired source. Furthermore, in the remaining of this work we assume that we can take the time-lag pairs from the ROS and therefore we can omit the noise terms in the equations, since the goal is extraction and not noise reduction.

From (3.1) it follows that the extraction vector $\mathbf{w}_1$ has to satisfy the following equation to perform the actual extraction:

$$\mathbf{w}_1\mathbf{A} = \alpha\mathbf{e}_1 \quad \forall \alpha \neq 0, \tag{3.2}$$

with $\mathbf{e}_1$ the standard basis row vector with a one at the first column and zeros at all other columns. For the full rank square mixing system, the extracting solution is given by:

$$\mathbf{w}_1 = \alpha\mathbf{e}_1 \left(\mathbf{A}\right)^{-1}. \tag{3.3}$$

From this result we can see that this solution is obtained by taking a scaled row vector of the inverse of the mixing system. This is only possible when the mixing system is a full-rank matrix and $\alpha \neq 0$. The solution of the over-determined mixing system can be found by using the pseudo inverse or by taking a subset of S sensors. Subsequently the remaining sensors can be used for noise reduction, but this is out of the scope of this paper. Therefore, in this paper we develop our method on the square mixing case. We will illustrate the solution in (3.3) in the following example.

Example 3.1. *Suppose we have a* 3×3 *mixing matrix given by:*

$$\mathbf{A} = \begin{bmatrix} \mathbf{a}^1 & \mathbf{a}^2 & \mathbf{a}^3 \end{bmatrix} = \begin{bmatrix} a^1_1 & a^2_1 & a^3_1 \\ a^1_2 & a^2_2 & a^3_2 \\ a^1_3 & a^2_3 & a^3_3 \end{bmatrix}.$$

Writing down the first row of the inverse of $\mathbf{A}$ *results in the desired extraction vector:*

$$\mathbf{w}_1 = \frac{\alpha}{\det \mathbf{A}} \begin{bmatrix} a^2_2 a^3_3 - a^3_2 a^2_3 & -(a^2_1 a^3_3 - a^3_1 a^2_3) & a^2_1 a^3_2 - a^3_1 a^2_2 \end{bmatrix},$$

and thus according to (3.2)*:*

$$\mathbf{w}_1\mathbf{A} = \begin{bmatrix} \alpha & 0 & 0 \end{bmatrix}.$$

Note that this is the desired extraction vector and that the solution $\mathbf{w}_1$ *is only depending on the undesired mixing columns* $\mathbf{a}^2, \mathbf{a}^3$. $\square$

It can be shown in general that the desired extraction vector is depending only on the undesired mixing columns:

$$\mathbf{w}_1 \perp \operatorname{span}\left(\mathbf{a}^2, \cdots, \mathbf{a}^S\right).$$

In future research, when for example a priori knowledge about the desired mixing column is known, we know that we cannot directly use that information on the extracting solution.

4. PROPOSED METHOD BASED ON SOS MATRIX STRUCTURE

In this section we propose a method to obtain the extraction vectors by writing the source correlation functions into the structure proposed in [1, 2], and subsequently performing a Generalized Eigenvalue Decomposition (GEVD).

Stacking the sensor correlation functions for every time-lag pair $[n,k] \in \Omega$ and all combinations of sensors in a special structure correlation matrix $\mathbf{C}^x$ of size $D^2 \times N$, we obtain:

$$\mathbf{C}^x \triangleq \begin{bmatrix} r^x_{11}[\Omega_1] & r^x_{11}[\Omega_2] & \cdots & r^x_{11}[\Omega_N] \\ r^x_{12}[\Omega_1] & r^x_{12}[\Omega_2] & \cdots & r^x_{12}[\Omega_N] \\ \vdots & \vdots & \ddots & \ddots \\ r^x_{DD}[\Omega_1] & r^x_{DD}[\Omega_2] & \cdots & r^x_{DD}[\Omega_N] \end{bmatrix},$$

with $\Omega_i = [n_i, k_i]\ \forall 1 \le i \le N$. We can also describe such a structure for the source correlation functions:

$$\mathbf{C}^s \triangleq \begin{bmatrix} r^s_{11}[\Omega_1] & r^s_{11}[\Omega_2] & \cdots & r^s_{11}[\Omega_N] \\ r^s_{22}[\Omega_1] & r^s_{22}[\Omega_2] & \cdots & r^s_{22}[\Omega_N] \\ \vdots & \vdots & \ddots & \ddots \\ r^s_{SS}[\Omega_1] & r^s_{SS}[\Omega_2] & \cdots & r^s_{SS}[\Omega_N] \end{bmatrix},$$

which is of size $S \times N$. Using the structure of these matrices, we can express the sensor correlation matrix $\mathbf{C}^x$ in terms of the mixing system $\mathbf{A}$ and the source correlation matrix $\mathbf{C}^s$:

$$\mathbf{C}^x = (\mathbf{A} \diamond \mathbf{A})\mathbf{C}^s = \mathbf{A}^2_\diamond \mathbf{C}^s, \tag{4.1}$$

where $\mathbf{A}^2_\diamond = \mathbf{A} \diamond \mathbf{A}$ is the second order Khatri-Rao product of the mixing system and $\mathbf{C}^s$ is a full rank matrix because of the assumption in (2.2). We now consider three cases, first of all if $N < S$, the extraction vector cannot be obtained, since:

$$\operatorname{rank}(\mathbf{C}^x) = N < S.$$

The second case is when $N = S$, then we can split the matrix $\mathbf{C}^x$ into S full rank sub-blocks:

$$\mathbf{C}^x = \begin{bmatrix} \mathbf{C}^x_1 \\ \vdots \\ \mathbf{C}^x_S \end{bmatrix}, \text{ with: } \mathbf{C}^x_i = \mathbf{A}\operatorname{diag}(\mathbf{a}_i)\mathbf{C}^s,$$

with $\operatorname{diag}(\cdot)$ a diagonal matrix with the elements of the input vector on the diagonal. If we now take 2 random linear combinations of the transpose of these sub-blocks, as already described for BI in [1], we can write the extraction problem as a generalized eigenvalue problem according to:

$$[\mathbf{W}, \mathbf{\Lambda}] = \operatorname{gevd}(\mathbf{\Gamma}_1, \mathbf{\Gamma}_2), \text{ with: } \mathbf{\Gamma}_j = \sum_{i=1}^{S} \xi^j_i (\mathbf{C}^x_i)^T, \tag{4.2}$$

where $(\boldsymbol{\xi}^1 \in \mathbb{R}^S) \neq (\boldsymbol{\xi}^2 \in \mathbb{R}^S)$ are two random vectors, $\mathbf{W}$ is a matrix containing the eigenvectors and $\mathbf{\Lambda}$ is a matrix containing the eigenvalues. Note that if the linear combinations of the sub-matrices remain full rank, then the eigenvectors of this problem are scaled rows of the inverse of the mixing system, thus all our solutions $\mathbf{w}_i$. This holds, because we can write from 4.2:

$$\sum_{i=1}^{S} \xi^1_i (\mathbf{C}^s)^T \operatorname{diag}(\mathbf{a}_i)(\mathbf{A})^T \boldsymbol{\mu} = \lambda \sum_{i=1}^{S} \xi^2_i (\mathbf{C}^s)^T \operatorname{diag}(\mathbf{a}_i)(\mathbf{A})^T \boldsymbol{\mu},$$

and this has the following S solutions for the eigenvalues λ:

$$\lambda_p = \frac{\sum_{i=1}^{S} a^p_i \xi^1_i}{\sum_{i=1}^{S} a^p_i \xi^2_i} \quad \forall 1 \le p \le S. \tag{4.3}$$

Now the S corresponding eigenvectors $\boldsymbol{\mu}^p$ are the transpose of the corresponding row vector of $(\mathbf{A})^{-1}$, thus the transpose of our desired solution $\mathbf{w}_i$. Note from (4.3) that the eigenvalue only contains information of the desired mixing column, while the extraction vector $\mathbf{w}_i = (\boldsymbol{\mu}^p)^T$ is independent of this mixing column. Therefore, it must be possible in future research to take a priori knowledge about the desired mixing column into account, to directly select the eigenvector-eigenvalue pair corresponding to the desired source.

To obtain more insight in the method, an example of the 2×2 extraction problem is given.

Example 4.1. *The special correlation matrix $\mathbf{C}^x$ for a 2×2 mixing system is given by:*

$$\mathbf{C}^x = \begin{bmatrix} a^1_1 a^1_1 & a^2_1 a^2_1 \\ a^1_1 a^1_2 & a^2_1 a^2_2 \\ a^1_2 a^1_1 & a^2_2 a^2_1 \\ a^1_2 a^1_2 & a^2_2 a^2_2 \end{bmatrix} \begin{bmatrix} r^s_{11}[k_1] & r^s_{11}[k_2] \\ r^s_{22}[k_1] & r^s_{22}[k_2] \end{bmatrix}.$$

In this case only 2 sub-blocks can be identified:

$$\mathbf{C}^x_1 = \mathbf{A} \begin{bmatrix} a^1_1 & 0 \\ 0 & a^2_1 \end{bmatrix} \mathbf{C}^s, \qquad \mathbf{C}^x_2 = \mathbf{A} \begin{bmatrix} a^1_2 & 0 \\ 0 & a^2_2 \end{bmatrix} \mathbf{C}^s.$$

When we choose $\boldsymbol{\xi}^j = \mathbf{e}^j$, with $j = 1, 2$, the following generalized eigenvalue problem is obtained from (4.2)*:*

$$(\mathbf{C}^x_1)^T \boldsymbol{\mu} = \lambda (\mathbf{C}^x_2)^T \boldsymbol{\mu},$$

where $\mathbf{C}^x_1$ and $\mathbf{C}^x_2$ are both full rank and unique because of (2.2) *and because $\mathbf{A}$ is full rank. The eigenvalues are now given by: $\lambda_1 = \frac{a^1_1}{a^1_2}$ and $\lambda_2 = \frac{a^2_1}{a^2_2}$ and the corresponding eigenvectors are transposed and scaled row vectors of the inverse mixing system.* □

In the final case where $N > S$, we first have to reduce the matrix $\mathbf{C}^x$. This reduction can be performed with help of the SVD. We can write:

$$\mathbf{C}^x = \mathbf{U}_x \mathbf{\Sigma}_x (\mathbf{V}_x)^T = \mathbf{U}_s \mathbf{\Sigma}_s (\mathbf{V}_s)^T + \mathbf{U}_\nu \mathbf{\Sigma}_\nu (\mathbf{V}_\nu)^T,$$

where also a decomposition in signal and noise space is made. It is known that the noise space eigenvalues ideally equal zero and the rank of the signal subspace equals S. If we now take the part $\mathbf{U}_s$, we have a linear combination of $\mathbf{A}^2_\diamond$, similar to (4.1), given by:

$$\mathbf{U}_s = \mathbf{A}^2_\diamond \mathbf{M},$$

where $\mathbf{M}$ a full rank matrix is of size $S \times S$. We can again identify the extraction vectors $\mathbf{w}_1$, by finding the generalized eigenvectors of 2 linear combinations of S sub-blocks of $\mathbf{U}_s$, thus:

$$\mathbf{U}_s = \begin{bmatrix} \mathbf{U}^x_1 \\ \vdots \\ \mathbf{U}^x_S \end{bmatrix}, \text{with: } \mathbf{U}^x_i = \mathbf{A}\operatorname{diag}(\mathbf{a}_i)\mathbf{M}.$$

Now solving the GEVD problem as defined in (4.2), with $\mathbf{C}^x_i$ replaced by $\mathbf{U}^x_i$, will give all extraction vectors in a matrix $\mathbf{W}$, which is the BSS solution. By choosing only one of the vectors, we have the extraction vector $\mathbf{w}_1$ of one of the sources.

5. SIMULATION RESULTS

By means of a simulation we will show the validity of the method. A mixture of 3 speech signals, measured with 3 sensors, is contaminated by mutually statistically independent white Gaussian noise sequences, all with a variance of $(\sigma^\nu)^2 = 1$. The source signals are 10000 samples of speech signals, sampled with 8 kHz and normalized such that $(\sigma^s)^2 = 1$. The Signal-to-Noise Ratio (SNR) is defined by:

$$\text{SNR} \triangleq \frac{\mathrm{E}\{||\mathbf{A}\mathbf{s}||^2\}}{\mathrm{E}\{||\boldsymbol{\nu}||^2\}} = \frac{S(\sigma^s)^2}{D(\sigma^\nu)^2},$$

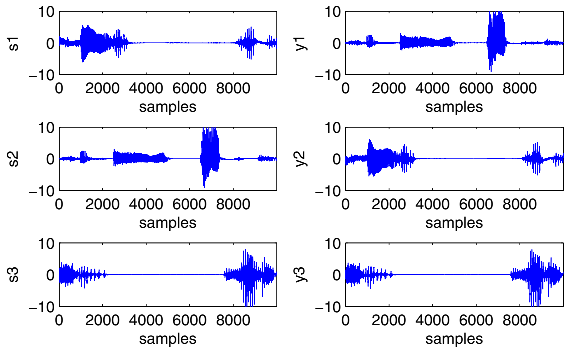

Fig. 2: Original source signals (left) and separated and permutated signals (right), with $\mathrm{SNR} = 0\,\mathrm{dB}$.

thus the SNR for our simulations equals $0\,\mathrm{dB}$. The mixing system, with normalized mixing columns, is given by:

$$\mathbf{A} = \begin{bmatrix} 0.6749 & 0.4082 & 0.8083 \\ 0.5808 & -0.8165 & 0.1155 \\ 0.4552 & -0.4082 & -0.5774 \end{bmatrix}.$$

The measured signals are split in 5 blocks of 2000 samples and we have taken the lags $1 \le k \le 5$, for every block. The ROS, with a size of $N = 25$, now becomes:

$$\Omega = \{(1,1), (1,2), \cdots, (1,5), (2,1) \cdots, (5,5)\}. \tag{5.1}$$

Since we have not used the lag $k = 0$ and we have white Gaussian noise sequences, we do not suffer from noise.

The developed method can be used to extract all sources individually. Since the focus has been on the extraction part, no performance for selecting the desired source can be measured, but we are able to separate all sources, thus we can use a Performance Index (PI) used in [2] for BSS, which measures the residual amount of mixing. If we define $\mathbf{T} \triangleq (\mathbf{W})^T\mathbf{A}$, we obtain the following PI:

$$\mathrm{PI} = (D)^2 \left(\sum_{i=1}^{D} \left\{ \frac{\sum_{j=1}^{D} |t_i^j|}{\max_k |t_i^k|} + \frac{\sum_{j=1}^{D} |t_j^i|}{\max_k |t_k^i|} - 2 \right\} \right)^{-1},$$

In Fig. 2 the original sources and the separated signals of one run are depicted. Notice that the extraction vectors are estimated on the noisy signals, while the actual extraction is performed on noise-free mixtures of the source, since the goal is not to perform noise reduction. The performance of this run is given by: $\mathrm{PI} = 43$. A histogram of the PI values for 5000 runs, with every time new noise sequences, is depicted in Fig. 3. A Gaussian approximation is made and we observe that PI behaves like a Gaussian function with a mean of 34 and a variance of $(\sigma^{\mathrm{PI}})^2 = 257$. This means that 78% of the estimations lie in the region: $18 \le \mathrm{PI} \le 50$.

6. CONCLUSIONS

We described the solution of the extraction problem for a square and over-determined mixing system. Based on SOS and well known mathematical tools like the SVD and eigenvalue decomposition, a method is developed to find the extracting solutions. By means of a simulation, the validity of the method is shown. It is also shown that with help of a priori information about the desired mixing column, an estimation of the generalized eigenvalue can be obtained. This can help future work to develop a method to find the desired source, with a minimum amount of a priori knowledge.

This work only forms a basis for the BSE problem and more investigation is required to deal with real-life situations. In future research, following important issues are some of the main problems which have to be resolved:

- The method should be extended to use a priori knowledge, which can be used to directly select the desired source. We already showed in (4.3) that information about the desired mixing column is embedded in the generalized eigenvalues;
- Besides the mixing column information, it should also be possible to use a priori statistical information about the desired source to directly find the extracting solution;
- Replacement of the mathematical tools by methods which are more suitable for online or realtime implementation;
- Development of a method to obtain the desired row vector from the pseudo inverse of the mixing matrix combined with noise reduction, in case of an over-determined system;
- The BSE problem should also be solved for the convolutive mixing case, where filters instead of scalars are used for the mixing, which is more realistic in acoustic applications.

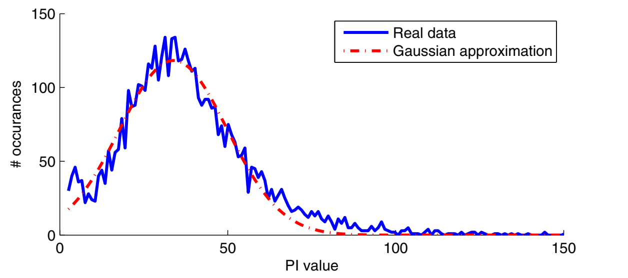

Fig. 3: Histogram of Performance Index (PI) with 5000 different noise sequences for $\mathrm{SNR} = 0\,\mathrm{dB}$.

7. REFERENCES

[1] J. van de Laar, *MIMO Instantaneous Blind Identification and Separation based on Arbitrary Order Temporal Structure in the Data*, Ph.D. thesis, Technische Universiteit Eindhoven, Nov. 2007.

[2] J. van de Laar, M. Moonen, and P. Sommen, "Mimo instantaneous blind identification based on second order temporal structure," *IEEE Trans. Signal Processing*, Accepted for publication in May 2008.

[3] A.-J. van der Veen, "Joint diagonalization via subspace fitting techniques," in *ICASSP '01: Proceedings of the Acoustics, Speech, and Signal Processing, 2001. on IEEE International Conference*, Washington, DC, USA, 2001, pp. 2773–2776, IEEE Computer Society.

[4] A. Belouchrani, K. Abed-Meraim, J. Cardoso, and E. Moulines, "A blind source separation technique using second-order statistics," 1997.

[5] Xi-Lin Li, "Sequential blind extraction adopting second-order statistics," *IEEE Signal Processing Lett.*, vol. 14, no. 1, pp. 58–61, Jan. 2007.