

A NEW APPROACH TO DIGITAL DIRECTIVITY CONTROL OF LOUDSPEAKERS LINE ARRAYS USING WAVE FIELD SYNTHESIS THEORY

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ABSTRACT

Loudspeakers line arrays are very common sound systems used in real environments as theatres, stadiums, cinemas, conference halls, etc, which synthesize the particular characteristics of a linear source. In the 80s a DSP technique to digitally control the sound field emitted by a linear source, based on filters and delay lines, has been introduced. In this paper we propose a new approach to digital pointing, based on Wave Field Synthesis (WFS). WFS is a new technique which allows the accurate reconstruction of the sound field emitted by sound sources. The proposed idea consists in using WFS to synthesize the sound field emitted by a linear source, positioned behind the loudspeakers line array. The advantage of this approach derives essentially from the fact that the linear source is virtual, so that the amplitude and the direction of the sound beam may be easily manipulated by changing length and orientation of the virtual source.

Index Terms— Wave Field Synthesis, Digital Directivity Control, Line Arrays

1. INTRODUCTION

A loudspeaker line array is a sound system constituted by a linear group of loudspeakers. The linear setup gives particular characteristics to the sound field, so that a line array is considered a very useful sound system in the *sound reinforcement* area. Many studies have been carried out on the design and use of vertical line array in theatres, concerts, cinemas, conference halls, etc.

A very important aspect of the sound field emitted by a vertical line array is referred to the possibility of pointing in a particular direction the main lobe of the sound beam by using digital signal processing techniques. In 1983 David G. Meyer proposed a method to control characteristics of a sound field emitted by a vertical line array [1, 2]. This work is the basis of all present implementations of the sound beam digital directivity control (DDC). The digital pointing can be obtained

without mechanical movements, so that the line array can be well integrated with the surrounding environment and corrections to the array inclination in order to change the pointing angle are avoided. Moreover, with a mechanical pointing only the sound energy emitted by the front of the array contributes to the direct sound field, while the one emitted by the back of the array contributes to the reverberant sound field. Instead, with a digital pointing both the front and the back of the array contribute to the direct sound field. A digital pointing may be obtained by modifying the signal amplitude and by using band-pass filters and delay lines. The signal amplitude can be enhanced or decreased even to zero to turn off some loudspeakers, thus changing the effective array length. Furthermore, it is possible to make the directivity independent from the signal frequency by applying to it appropriate band-pass filters. Finally, the shape of the sound field can be modified using suitable delay lines. Combining these three signal controlling actions, the sound system achieves particular emission characteristics.

In this paper we propose to apply the possibility of controlling the amplitude and the direction of the sound beam by using Wave Field Synthesis (WFS) theory [3, 4, 5]: the idea is to derive the sound field of a virtual linear source, positioned behind the loudspeakers array, by using the WFS formulae. In this way, it is possible to control the main lobe amplitude and the direction of the sound beam by changing the length and the orientation of the virtual linear source.

The paper is organized as follows: firstly, the mathematical model of the sound field emitted by a linear source and the theoretical basis of the WFS systems will be reviewed; secondly, the way to control the amplitude and the direction of the sound beam reproduced by the Wave Field Synthesis will be described. Finally, simulation results will be reported to validate the proposed approach.

2. LOUDSPEAKERS LINE ARRAYS THEORY

The sound pressure emitted by a linear source of length L can be calculated as the sum of the contributions of point sources

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[6, 7]. In formulae:

$$p(\vec{r}, t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{A(l)e^{-j[kr(l)+\varphi(l)]}}{r(l)} dl \quad (1)$$

where $A(l)$ and $\varphi(l)$ are the amplitude function and the phase function along the linear source axis respectively, k is the wave number and $r(l)$ is the distance between the segment dl and the sound pressure observation point. This relation can be simplified considering the assumption that the sound pressure is measured in far-field conditions, that is the distance r between the line array and the observation point satisfies the following relation:

$$r \gg L \quad (2)$$

In this way, every distances between each segment dl and the sound pressure observation point can be considered parallel, so that the sound pressure becomes:

$$p(\vec{r}, t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{A(l)e^{-j[kr+\varphi(l)]}}{r} dl \quad (3)$$

Assuming $A(l) = A$ and $\varphi(l) = 0$ the sound pressure for a uniform linear source is obtained and the previous equation is furthermore simplified:

$$p(r, t) = \int_{-\frac{L}{2}}^{\frac{L}{2}} \frac{Ae^{-jkr}}{r} dl \quad (4)$$

For uniform linear sources it is important to consider the quarter power angle: this angle corresponds to an amplitude of $-6dB$ normalized to its maximum. It is defined as following (in degrees) [7]:

$$\theta_{-6dB} \approx 68.8 \frac{\lambda}{L} = \frac{68.8}{n} \quad (5)$$

where λ is the wave length and n is the ratio between the linear source length and the wave length. This parameter is commonly used to estimate the amplitude of the emission main lobe. As an example, the emission main lobe has an amplitude of about 23 degrees for $n = 3$.

3. WAVE FIELD SYNTHESIS THEORY

The theoretical basis of Wave Field Synthesis are given by Kirchhoff-Helmholtz integrals which state: *the sound field inside a volume V enclosed by a surface S can be calculated if the pressure field and normal particle velocity due to the primary sources on the enclosing surface are known* [3].

In literature it can be seen that, in order to reproduce the desired sound field with a linear distribution of loudspeakers, a particular surface constituted by a plane S_0 and a semi-sphere S_1 can be considered. Therefore, the sound pressure field is defined as follows (*Rayleigh I integral*):

$$P(\vec{r}, \omega) = \frac{1}{2\pi} \int_{S_0} jk\rho_0 c V_n(\vec{r}_S, \omega) \frac{e^{-jk|\vec{r}_S - \vec{r}|}}{|\vec{r}_S - \vec{r}|} dS \quad (6)$$

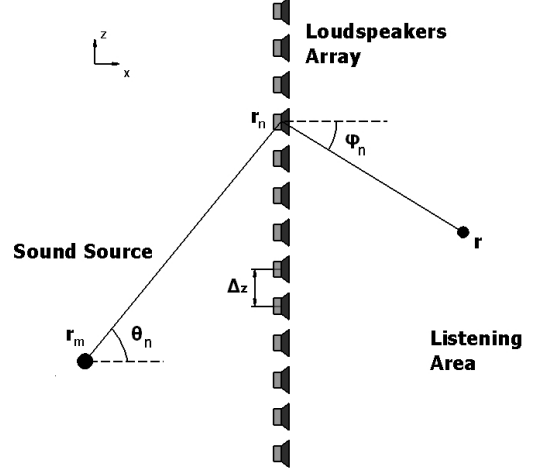


Fig. 1. Geometry used for the driving function calculation relative to linear array.

where \vec{r} and \vec{r}_S denote the generic position inside V and on S respectively, ω is the angular frequency, k is the wave number, c is the sound velocity, ρ_0 is the medium density and V_n is the particle velocity normal to S .

In a practical implementation the secondary sources are loudspeakers distributed along a line. Therefore, a discrete distribution instead of a continuous one has to be considered and the surface S_0 has to be reduced to a line. Hence, taking into account the link between the particle velocity and the sound pressure [8], the discrete form of Rayleigh integral is obtained:

$$P(\vec{r}, \omega) = \left(\frac{jk}{2\pi} + \frac{1}{2\pi r} \right) \sum_{i=1}^{N_L} P(\vec{r}_i, \omega) \frac{e^{-jk|\vec{r}_i - \vec{r}|}}{|\vec{r}_i - \vec{r}|} \Delta z \quad (7)$$

where P is the sound pressure, z is the array axis, Δz is the distance between two adjacent loudspeakers, \vec{r}_i is the position of the i -th loudspeaker and N_L is the number of loudspeakers.

To reproduce the desired sound field it is needed to find an appropriate driving function for each loudspeaker. With reference to Fig. 1, the driving functions can be derived from (7) as follows

$$P(\vec{r}, \omega) = \sum_{i=1}^{N_L} Q(\vec{r}_i, \omega) D(\varphi_i, \omega) \frac{e^{-jk|\vec{r}_i - \vec{r}|}}{|\vec{r}_i - \vec{r}|} \Delta z \quad (8)$$

where $Q(\vec{r}_i, \omega)$ and $D(\varphi_i, \omega)$ are the driving function and the directivity function relative to the i -th loudspeaker. The driving functions needed to obtain the sound field produced by a monopole at position \vec{r}_m can be calculated solving the following equation

$$S(\omega) \frac{e^{-jk|\vec{r} - \vec{r}_m|}}{|\vec{r} - \vec{r}_m|} = \sum_{i=1}^{N_L} Q(\vec{r}_i, \omega) D(\varphi_i, \omega) \frac{e^{-jk|\vec{r}_i - \vec{r}|}}{|\vec{r}_i - \vec{r}|} \Delta z \quad (9)$$

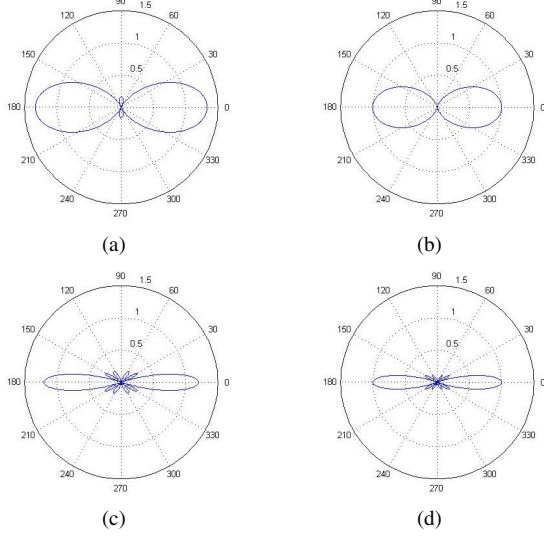


Fig. 2. Polar responses considering linear sources composed of finite N_S monopole sources (Fig. 2(a) and Fig. 2(c)) and ideal linear sources (Fig. 2(b) and Fig. 2(d)). The upper figures have been obtained for $n = 1$ ($N_S = 4$ in the not ideal case). The lower figures have been obtained for $n = 3$ ($N_S = 6$ in the not ideal case).

where $S(\omega)$ is the signal to be reproduced in the frequency domain. After a substantial mathematical manipulation [3] the driving function is obtained:

$$Q(\vec{r}_i, \omega) = S(\omega) \frac{\cos \theta_i}{D_i(\theta_i, \omega)} \sqrt{\frac{jk}{2\pi}} \sqrt{\frac{|x_0 - x_i|}{|x_0 - x_m|}} \frac{e^{-jk|\vec{r}_i - \vec{r}_m|}}{|\vec{r}_i - \vec{r}_m|}. \quad (10)$$

In the following, without loss of generality, the loudspeaker will be viewed as an omnidirectional source. Therefore D will be assumed equal to 1.

4. DIRECTIVITY CONTROL USING WFS APPROACH

The proposed approach is an application of WFS theory to digital directivity control problem. It consists in the synthesis of the sound field emitted by a linear source positioned behind the loudspeakers array through WFS approach. The advantage is that linear sources of arbitrary length and orientation can be used, given the fact that they are virtual sources, so that the sound beam can be freely controlled. However, the loudspeakers array used for the reproduction imposes frequency bounds, because it has a fixed length. Considering an array of length Z , the minimum and maximum frequencies that can be reproduced are given by the following equations:

$$f_{min} = n \frac{c}{Z} \quad (11)$$

$$f_{max} = \frac{c}{dz} \quad (12)$$

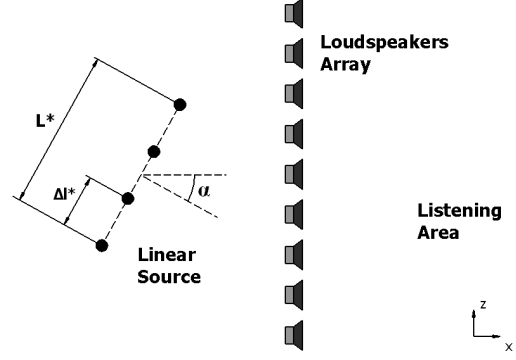


Fig. 3. Virtual source position relative to the loudspeakers line array ($N_S = 4$).

where dz is the distance between two adjacent elements of the array and n is defined as in section 2. Therefore, the minimum and maximum frequencies which can be reproduced with a particular sound beam depend on the length of the loudspeakers array and the distance between two adjacent elements respectively.

The linear source can be considered as a combination of monopole sources. Assuming a frequency f^* , the length of the linear source, which ensures the desired main lobe amplitude, is defined as follows:

$$L^* = n \frac{c}{f^*} \quad (13)$$

The distance between two adjacent elements is calculated by using the following relation:

$$dl^* = \frac{L^*}{N_S - 1} \quad (14)$$

where N_S is the number of monopole sources. Increasing the value of N_S , the main lobe becomes similar to the ideal case and the presence of secondary lobes is minimized. However, $N_S = 4$ and $N_S = 6$ give a good approximation for a main lobe of about 70 ($n = 1$) and 25 ($n = 3$) degrees respectively, as shown in Fig. 2.

In order to point the sound beam in the desired direction, an appropriate translation and rotation of the linear source reference axis has to be performed (Fig. 3).

Finally, the sound field of the linear source, composed of N_S elements, is obtained through the physical loudspeakers array of length Z , driven by appropriate WFS driving functions. Each driving function is calculated as the sum of the N_S monopole sources contributes (10).

In a real environment, an increasing of the quarter power angle of about 30% can be considered not relevant, so that the described approach results valid in a range of frequency comprised from $f^*/\sqrt{2}$ and f^* (for $n = 3$ the increasing of the quarter power angle is about 6 degrees). Therefore, a

generic input signal has to be divided into appropriate subbands. Once the f_{min} has been fixed by the physical loudspeakers array, it is taken as reference for the subband division, which is hence performed according to the following relation:

$$B_{q+1} = B_q \cdot \sqrt{2} \quad (15)$$

starting from $B_0 = f_{min}$ up to the maximum frequency and each q -th subband is included in the range $[B_q; B_{q+1}]$. The proposed subband division has to be considered as an example: it is possible to choose more constrained band limits, in order to obtain a less increase of the quarter power angle. In a practical implementation, the subband division may be performed through a simple crossover network [9].

Simulations of the proposed approach for the control of the sound beam directivity have been carried out. A line array of length $Z = 0.921m$, composed of 8 loudspeakers of $0.102m$ diameter ($f_{min} = 415Hz$ and $f_{max} = 2900Hz$), has been considered for the sound field synthesis. In order to obtain a quarter power angle of about 23 degrees, $n = 3$ has been chosen. Fig. 4 shows the sound field related to a signal of frequency $f = 1400Hz$ with an orientation of 30 degrees. The sound field has been reproduced using the proposed approach and classic Digital Directivity Control. It can be seen that the presence of secondary lobes is less visible in the latter case.

5. CONCLUSION

In this paper we have described a new method for digitally control the amplitude and the direction of a sound beam. It is based on the subband division of the input signal and on the design of an appropriate virtual linear source, whose length depends on each subband limits. The sound field of each linear source is reconstructed applying WFS theory. The advantage of this approach is that the length of each linear source can be chosen in a proper way in order to obtain a minimum presence of secondary lobes of the sound beam. Moreover, it can be easily pointed in the desired direction through a simple transformation. Simulations results confirm the effectiveness of the proposed approach.

6. REFERENCES

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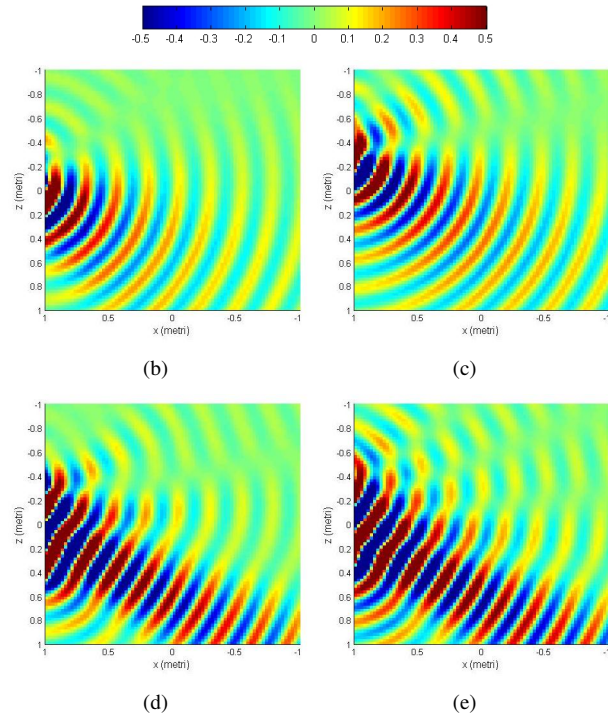


Fig. 4. Sound field reconstruction with a direction of 30 degrees and a frequency of 1400 Hz. Fig. 4(b) and Fig. 4(c) has been obtained for $n = 1$ with and without WFS respectively. Fig. 4(d) and Fig. 4(e) has been obtained for $n = 3$. WFS tests have been carried out considering a linear source length optimally designed for a frequency of 1660 Hz.

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