NLMS-TYPE SYSTEM IDENTIFICATION OF MISO SYSTEMS WITH SHIFTED PERFECT SEQUENCES

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ABSTRACT

This paper addresses the fundamental problem of multi-channel system identification given the *multiple input* and *single output* signals of a MISO system. The presented approach is based on the *normalized least mean square* (NLMS) algorithm in combination with a set of special excitation signals, constructed from so called *perfect sequences* (PSEQs). The new excitation strategy opens up the possibility to uniquely identify the true impulse responses of multiple channels in a simple and efficient way from a single-time recording. The method can be applied to any number of channels or system lengths. Due to its fast tracking property, this new approach allows the real-time acquisition of time-variant impulse responses. These can be used, e.g., in simulations for the evaluation of stereophonic acoustic echo cancellation algorithms under real-world conditions or for the calibration of multi-channel loudspeaker systems. Furthermore, the method allows an identification of linear channels of any kind, e.g., radio or acoustic, and can easily be extended to MIMO (e.g. wireless) transmission.

Index Terms— System identification, NLMS algorithm, perfect sequences, adaptive filters, multi-channel systems

1. INTRODUCTION

The problem of system identification has been addressed in many publications. For the measurement of acoustic transfer functions, e.g., the so called *fast M-sequence* transform was developed [1–3] based on the cross-correlation between a special stimulus signal and its system response. In [4,5] we introduced an alternative technique, which relies on an NLMS-type algorithm [6] excited by a so called *perfect sequence* (PSEQ) [7,8], i.e., a special periodically repeated pseudo noise signal. With this deterministic periodic excitation signal the NLMS algorithm is capable to identify a linear noiseless system within one period.

In this paper we develope a strategy of how to generalize this basic concept of system identification to a multi-channel system. The main question is how to construct optimal excitation signals to identify the different paths of a *multiple input – single output* (MISO) system.

The typical problem of multi-channel system identification as needed, e.g., in stereophonic acoustic echo cancellation [9, 10] or multi-channel surround sound simulation is the non-zero cross-correlation between the excitation signals. As a result the adaptive filters often don't converge on the *true* system impulse responses or show poor convergence speed. Given only a single measured reference signal, adequate excitation signals are required to identify the *true* impulse response of each channel.

In Sec. 2 we first introduce the basic terms and principles of the NLMS-type identification algorithm with PSEQ excitation. In the main part of this paper, in Sec. 3, we generalize this concept to the multi-channel case and present a technique to construct *optimal* excitation signals for multiple channels. Finally, in Sec. 4 we will verify our theoretical results with simulations.

2. SYSTEM IDENTIFICATION WITH PSEQS

The discrete time model in Fig. 1 depicts a system for the identification of an unknown linear transmission system by means of an adaptive filter $\mathbf{h}(k)$. The unknown transmission path is represented by the impulse response $\mathbf{g} = (g_0, g_1, \ldots, g_{N-1})^T$ of length N. The influence of environmental noise on the adaptation process can be taken into account by adding a non-zero noise signal $n(k)$ to the output of the unknown system $\mathbf{g}$.

The adaptation of the digital transversal filter is driven by the NLMS algorithm, i.e., the weights of the adaptive filter are controlled by the recursion

$$\mathbf{h}(k+1) = \mathbf{h}(k) + \alpha \frac{e(k)\,\mathbf{p}(k)}{||\mathbf{p}(k)||^2} \tag{1}$$

with stepsize α and the error signal $e(k)$

$$e(k) = [\mathbf{g} - \mathbf{h}(k)]^T\ \mathbf{p}(k) + n(k)\,. \tag{2}$$

The vectors and the squared vector norm are given by

$$\mathbf{h}(k) = (h_0(k),\, h_1(k),\, \ldots,\, h_{N-1}(k))^T\ ,$$
$$\mathbf{p}(k) = (p(k),\, p(k-1),\, \ldots,\, p(k-N+1))^T\ ,$$
$$||\mathbf{p}(k)||^2 = \mathbf{p}^T(k)\,\mathbf{p}(k)$$

each of length N.

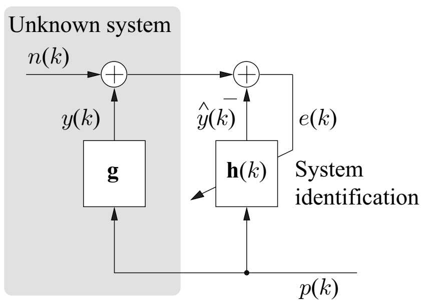

Fig. 1. Single-channel system identification with PSEQs.

As the convergence properties of the NLMS algorithm depend on the correlation properties of the excitation signal $p(k)$, *optimal* adaptation of the NLMS algorithm can be achieved if N consecutive excitation vectors $\mathbf{p}(k),\ \mathbf{p}(k-1),\ \ldots\mathbf{p}(k-N+1)$ are orthogonal to each other [4]. This requirement can be fullfilled by the use of a PSEQ of length N periodically applied as excitation signal. A PSEQ has a periodic impulse-like autocorrelation function

$$\tilde{R}_{pp}(\lambda) = \sum_{i=0}^{N-1} p(i)\,p(\lambda+i) \tag{3}$$

$$= \begin{cases} ||\mathbf{p}(\lambda)||^2 & \text{for } \lambda \bmod N = 0 \\ 0 & \text{else}\,, \end{cases} \tag{4}$$

i.e., the cross-correlation of vector $\mathbf{p}(k)$ with all phase-shifted versions $\mathbf{p}(k \pm \lambda)$, $(\lambda \bmod N \neq 0)$ is zero. Various methods exist to construct (e.g., ternary) PSEQs. It is of importance that PSEQs are available for a sufficient variety of lengths as, e.g., the synthesis of adequate PSEQs according to [8] is possible for all lengths $N_p = q+1$ with $q = \rho^k$, $\rho > 2$ prime, $k \in \mathbf{N}$.

In order to visualize the effect of a PSEQ excitation, we apply a periodic PSEQ and a spectrally white noise signal as input signals $p(k)$ to the system given in Fig. 1. The quality is evaluated in terms of the (logarithmic) *system distance*

$$\frac{D(k)}{\text{dB}} = 10\ \lg \frac{||\mathbf{g} - \mathbf{h}(k)||^2}{||\mathbf{g}||^2}\,, \tag{5}$$

also known as *misalignment*. Fig. 4-a shows the results for both stimulus signals in case that

- the unknown system is linear and time-invariant (LTI), except a sudden change at $k = 4000$,
- the system is noiseless, i.e., $n(k) = 0$,
- the length of $\mathbf{h}(k)$ is "sufficiently long" with respect to the unknown system $\mathbf{g}$, and
- the length of $\mathbf{h}(k)$ equals the period of the PSEQ.

For these assumptions a stepsize of $\alpha = 1$ provides the best results and the accuracy of the identification process is only limited by the available computational precision.

Fig. 4-a confirms that in the continuous adaptation process after a sudden change of the unkown system $\mathbf{g}$ at $k = 4000$ only N iterations are needed for a complete identification within computational precision. Due to the empty filter states in the beginning of the simulation the adaptation process takes $2N$ iterations.

The direct comparison with the system distance achieved with a white noise excitation emphasizes how the NLMS benefits from the excitation with deterministic PSEQs. Let us consider, for instance, that we aim at a system distance of $D(k) = -20$ dB. At the point where the system distance for a PSEQ excitation meets -20 dB, the corresponding curve for a white noise stimulation shows only a system distance of -6.5 dB. For the latter the identification process takes 3.5 times longer to reach -20 dB.

In practice we have to deal with time-variant systems, physical impulse responses of infitive length, and environmental noise $n(k) \neq 0$. As a result, the generated set of coefficients $\mathbf{h}(k)$ does normally not match exactly the actual impulse response $\mathbf{g}$. These aspects relevant for practical applications, e.g., the influence of noise or the tracking properties in case of time-variant systems, have been extensively discussed in [4, 5].

Another interesting aspect of this concept is that it can be extended to the multi-channel case. In the following section we will focus on how to generalize the NLMS-type system identification approach with PSEQs from the single- to the multi-channel case.

3. MULTI-CHANNEL SYSTEM IDENTIFICATION

For the multi-channel system identification we consider a set-up according to Fig. 2 with M parallel unknown systems $\mathbf{g}^{(1)}$, $\mathbf{g}^{(2)}$, $\ldots$, $\mathbf{g}^{(M)}$. The task is to uniquely identify the *true* impulse responses by adapting the digital filters $\mathbf{h}^{(1)}(k)$, $\mathbf{h}^{(2)}(k)$, $\ldots$ $\mathbf{h}^{(M)}(k)$ given only one error signal $e(k)$. The multi-channel NLMS algorithm is given by

$$\mathbf{h}^{(1)}(k+1) = \mathbf{h}^{(1)}(k) + \frac{\alpha\, e(k)\, \mathbf{p}^{(1)}(k)}{E_p(k)} \tag{6}$$

$$\mathbf{h}^{(2)}(k+1) = \mathbf{h}^{(2)}(k) + \frac{\alpha\, e(k)\, \mathbf{p}^{(2)}(k)}{E_p(k)} \tag{7}$$

$$\vdots$$

$$\mathbf{h}^{(M)}(k+1) = \mathbf{h}^{(M)}(k) + \frac{\alpha\, e(k)\, \mathbf{p}^{(M)}(k)}{E_p(k)} \tag{8}$$

and

$$E_p(k) = ||\mathbf{p}^{(1)}(k)||^2 + ||\mathbf{p}^{(2)}(k)||^2 + \ldots + ||\mathbf{p}^{(M)}(k)||^2$$

with all vectors of length N. The M input signals are denoted by $p^{(1)}(k)$, $p^{(2)}(k)$, $\ldots$, $p^{(M)}(k)$ and have still to be defined. The indices $^{(1)}$, $^{(2)}$, $\ldots{}^{(M)}$ denote the channel index of the signals, systems, and system distances. The error signal results in

$$e(k) = \mathbf{p}^{(1)^T}(k) \cdot \left[\mathbf{g}^{(1)} - \mathbf{h}^{(1)}(k)\right] +$$

$$\mathbf{p}^{(2)^T}(k) \cdot \left[\mathbf{g}^{(2)} - \mathbf{h}^{(2)}(k)\right] + \tag{9}$$

$$\vdots \tag{10}$$

$$\mathbf{p}^{(M)^T}(k) \cdot \left[\mathbf{g}^{(M)} - \mathbf{h}^{(M)}(k)\right] + n(k)\,. \tag{11}$$

In the next step we introduce a PSEQ $\check{p}(k)$ with the enlarged period length $N_p = M \cdot N$. For the first channel we use $\check{p}(k)$ as excitation signal, while for all other channels adequately phase-shifted versions of the PSEQ are applied according to

$$p^{(1)}(k) = \check{p}(k) \tag{12}$$

$$p^{(2)}(k) = \check{p}(k-N) \tag{13}$$

$$\vdots$$

$$p^{(M)}(k) = \check{p}(k-(M-1)\,N)\,. \tag{14}$$

resulting in a special *periodic multi-phase excitation*. Below ' $\check{p}$ ' will always refer to PSEQs of period $N_p = M \cdot N$.

Now we will prove that this set of shifted PSEQs represent the *optimal* excitation signals $p^{(1)}(k)$, $p^{(2)}(k)$, $\ldots$, $p^{(M)}(k)$ for the multi-channel case. The NLMS recursions according to (6)-(8) change to

$$\mathbf{h}^{(1)}(k+1) = \mathbf{h}^{(1)}(k) + \frac{\alpha\, e(k)\, \check{\mathbf{p}}(k)}{E_p} \tag{15}$$

$$\mathbf{h}^{(2)}(k+1) = \mathbf{h}^{(2)}(k) + \frac{\alpha\, e(k)\, \check{\mathbf{p}}(k-N)}{E_p} \tag{16}$$

$$\vdots \tag{17}$$

$$\mathbf{h}^{(M)}(k+1) = \mathbf{h}^{(M)}(k) + \frac{\alpha\, e(k)\, \check{\mathbf{p}}(k-(M-1)\,N)}{E_p} \tag{18}$$

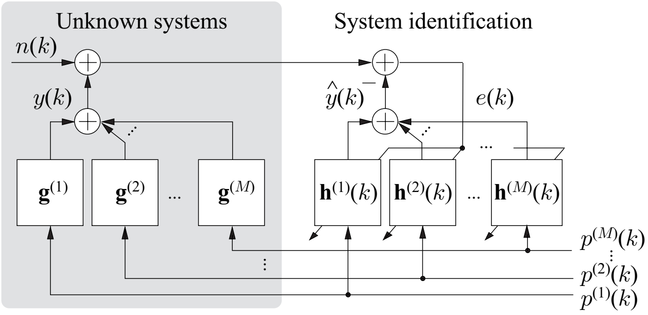

Fig. 2. Multi-channel system identification with PSEQs.

with the constant normalization factor

$$E_p = ||\check{\mathbf{p}}(k)||^2 + ||\check{\mathbf{p}}(k-N)||^2 + \ldots + ||\check{\mathbf{p}}(k-(M-1)\,N)||^2 = \sum_{i=0}^{MN-1} \check{p}^2(i)\,.$$

Note that all vectors are still of dimension N. Thus, each excitation vector $\check{\mathbf{p}}(k)$, $\check{\mathbf{p}}(k-N)$, $\ldots$, $\check{\mathbf{p}}(k-(M-1)\,N)$ contains only the Mth-part of the underlying PSEQ, i.e.,

$$\check{\mathbf{p}}(k) = (\check{p}(k),\, \check{p}(k-1),\, \ldots,\, \check{p}(k-N+1))^T$$
$$\check{\mathbf{p}}(k-N) = (\check{p}(k-N),\, \ldots,\, \check{p}(k-2N+1))^T$$
$$\vdots$$
$$\check{\mathbf{p}}(k-(M-1)N) = (\check{p}(k-(M-1)N),\, \ldots,\, \check{p}(k-MN+1))^T.$$

In order to prove that $p^{(1)}(k)$, $p^{(2)}(k)$, $\ldots$, $p^{(M)}(k)$ as defined in (12)-(14) fulfill the requirements of *optimal* excitation signals for a system with M channels, we transform the multi-channel system given in Fig. 2 into an equivalent single-channel system. For this reason we define new combined vectors

$$\mathbf{g}_{|MN} = \left(\mathbf{g}^{(1)},\, \mathbf{g}^{(2)},\, \ldots,\, \mathbf{g}^{(M)}\right)^T \tag{19}$$

$$\mathbf{h}(k)_{|MN} = \left(\mathbf{h}^{(1)}(k),\, \mathbf{h}^{(2)}(k),\, \ldots,\, \mathbf{h}^{(M)}(k)\right)^T \tag{20}$$

$$\check{\mathbf{p}}(k)_{|MN} = (\check{\mathbf{p}}(k),\, \check{\mathbf{p}}(k-N),\, \ldots,\, \check{\mathbf{p}}(k-(M-1)\,N))^T \tag{21}$$

each of length $N_p = M \cdot N$. Due to the special *periodic multi-phase excitation* the system reactions $y(k)$ and $\hat{y}(k)$ can thus be formulated according to

$$y(k) = \mathbf{g}^{(1)^T}\mathbf{p}^{(1)}(k) + \mathbf{g}^{(2)^T}\mathbf{p}^{(2)}(k) + \ldots + \mathbf{g}^{(M)^T}\mathbf{p}^{(M)}(k) = \mathbf{g}^T_{|MN} \cdot \check{\mathbf{p}}(k)_{|MN} \tag{22}$$

$$\hat{y}(k) = \mathbf{h}^T(k)_{|MN} \cdot \check{\mathbf{p}}(k)_{|MN}\,. \tag{23}$$

Exploiting (19)-(23), we transform the parallel filter structure of Fig. 2 into a serial structure according to Fig. 3 assuming that the transmission systems $\mathbf{g}^{(1)}$, $\mathbf{g}^{(2)}$, $\ldots$, $\mathbf{g}^{(M)}$ can be modelled by direct form filters. In this reorganized system the identification process is defined by

$$\mathbf{h}(k+1)_{|MN} = \mathbf{h}(k)_{|MN} + \frac{\alpha\, e(k)\, \check{\mathbf{p}}(k)_{|MN}}{||\check{\mathbf{p}}(k)_{|MN}||^2} \tag{24}$$

$$e(k) = \check{\mathbf{p}}^T(k)_{|MN} \cdot \left[\mathbf{g}_{|MN} - \mathbf{h}(k)_{|MN}\right] + n(k)\,. \tag{25}$$

Except for the initialization phase, the systems of Fig. 2 and Fig. 3 are equivalent due to the periodicity and the shift between the M input signals according to (12)-(14). While in the system given in Fig. 2 N iterations are needed to fill all filter states, in the serialized system (Fig. 3) N_p iterations are required. As we use the serialized system only for proving theoretically the optimality of the excitation the difference during the initialization is irrelevant.

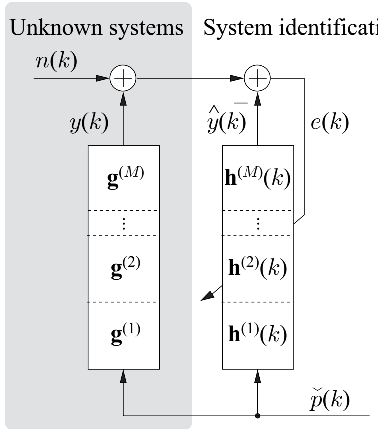

Fig. 3. Equivalent multi-channel system in serial structure, valid only for the special *periodic multi-phase excitation*.

With the reorganisation of the system we reduced the multi-channel case to the known single-channel problem with all dimensions M-times as long. As this problem has been solved in Sec. 2 we also can conclude that with a choice of $p^{(1)}(k)$, $p^{(2)}(k)$, $\ldots$, $p^{(M)}(k)$ according to (12)-(14) *optimal* excitation signals for the multi-channel case can easily be generated. Thus, the excitation of the M channels with shifted versions of the underlying basic PSEQ $\check{p}(k)$ allows the NLMS algorithm to uniquely identify the *true* impulse responses $\mathbf{g}^{(1)}$, $\mathbf{g}^{(2)}$, $\ldots$, $\mathbf{g}^{(M)}$ within one period N_p.

4. SIMULATION RESULTS

In order to verify above considerations we investigate different excitation strategies for the multi-channel system of Fig. 2. We oppose the results for a PSEQ and a white noise excitation with the assumptions according to Sec. 2. Fig. 4 illustrates the results for different numbers of channels in terms of the system distance of the first channel $D^{(1)}$. The results of the other channels look alike and thus are not considered in the plots. In the three examples of Fig. 4 identify systems with an order of approximately $N \approx 380$ are identified by means of the introduced algorithm. Consequently, the corresponding periods of the applied PSEQs differ.

Obviously, the algorithm is capable to perfectly separate and identify the impulse responses of the different channels within computational precision, which is due to the special orthogonality features of the excitation signals, i.e., each excitation signal has an ideal impulse-like auto-correlation function, and a zero cross-correlation to any other signal out of the set.

Especially, the comparison with the results obtained for white noise reflects the benefits of the set of shifted PSEQs. Note that for white noise the gradients of the system distances in Fig. 4 become less steep with increasing number of channels, as the effective cross-correlations between the white noise signals of the parallel channels are not zero. To pick up the example from Sec. 2 we can observe that in the multi-channel case with a white noise excitation it takes even 5 times longer than with shifted PSEQs to reach -20 dB.

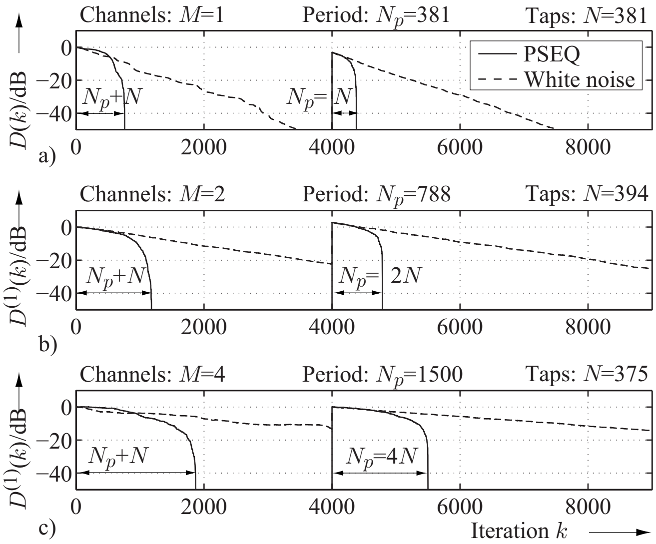

Fig. 4. System distances for PSEQ and white noise excitation for different multi-channel systems; all $\mathbf{g}^{(\nu)}$, $\nu \in \{1, \ldots, M\}$ are time-invariant, except a sudden change at $k = 4000$, $n(k) = 0$, $\alpha = 1$, comparable lengths N for all three cases.
a) Single-channel case, $N_p = N = 381$
b) Two-channel case, $N_p = 2N = 788$, similar results for $D^{(2)}(k)$
c) Four-channel case, $N_p = 4N = 1500$, similar results for $D^{(2)}(k)$, $D^{(3)}(k)$, and $D^{(4)}(k)$

5. CONCLUSIONS

The identification of one or more linear system(s) by means of an adaptive filter has a wide range of possible applications, e.g., in the area of multi-dimensional sound reproduction, room acoustics, or high quality videoconferencing. For the research, design and development of relevant algorithms knowledge about the nature of these channels is needed.

In this paper we proposed a new concept of multi-channel system identification with a special set of shifted PSEQs as excitation signals for the different channels. The use of PSEQ stimulus signals for the multi-channel case is motivated by the fact that they represent the *optimal* excitation signals for the single-channel case. Thus, starting from the well-known approach for one channel we developed a strategy to construct a set of *optimal* excitation signals for all parallel channels with one PSEQ. In theory and via simulations we have shown that the application of a set of shifted PSEQs to the inputs of a MISO system allows the identification of a linear, time-invariant, and noiseless multi-channel system within computational precision. In other words, with one measurement of an arbitrary number of channels the approach allows the identification and separation of all unkown systems in parallel.

In practical applications we have to deal with non ideal systems, i.e., systems might be time-variant, of infinite length, and/or environmental noise might occur which affects the identification process. However, as a result of its convergence speed also in these suboptimal conditions the proposed algorithm shows significantly better results than with white noise.

We use the identification algorithm, e.g., to track the fluctuations of time-variant impulse responses. Therefore, consecutive sets of coefficients can be used to simulate reproducibly real, time-variant transmission links, e.g., room impulse responses [11] for the design of stereophonic echo cancellation algorithms. In a medical application we use the novel approach to investigate the dynamic behavior of the Eustachian tube function [12, 13]. With a two-channel measurement prototype, e.g., we visualize the transmission links between the two nostrils and the ear as a function of time.

6. REFERENCES

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