OPTIMAL AZIMUTHAL STEERING OF A FIRST-ORDER SUPERDIRECTIONAL MICROPHONE RESPONSE

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ABSTRACT

An azimuth steerable first-order superdirectional microphone response can be constructed by combination of three eigenbeams (monopole and two orthogonal dipoles) and applying standard signal processing techniques. The main-lobe can be steered to a certain angle on the 2D plane, such that the desired signal is captured. Besides steering, the directional pattern can be adjusted in such a way that either a point-interferer coming from an other angle is rejected or that we get an optimal rejection of diffuse noise. However, when we want to reject both a point-interferer and obtain the best possible diffuse noise reduction, neither of these methods give optimal results. In this paper, an optimal steering method is derived to construct the first-order directional response in such a way that the point-interferer is removed with the best possible diffuse noise reduction.

Index Terms— Microphone arrays, null steering, superdirective beamformer.

1. INTRODUCTION

In applications such as hands-free communication and voice control systems, the microphone signal is corrupted by (diffuse) background-noise and reverberation. To reduce the amount of noise and reverberation, we can use a microphone array and apply beamforming techniques to steer the main-lobe of a beam towards the desired (speech) signal. For small microphone arrays, where the wavelength of the sound of interest is much larger than the size of the array, additive beamformers like delay-and-sum are not able to obtain a sufficient directivity, as the beamwidth deteriorates for these wavelengths [1], [2]. A common method to obtain an improved directivity is to apply superdirective beamforming techniques. To obtain this superdirectivity, beamforming techniques require filters with asymmetrical filter coefficients [3]. Basically, this asymmetrical filtering corresponds to subtraction of signals, like in the delay-and-subtract techniques [4] [5] or by taking spatial derivatives of the sound pressure field [6] [7] (discussed in Section 2). Although the directivity can be improved by such asymmetrical filtering, it is also known that this is obtained at the cost of robustness, such as increased sensitivity for white (sensor) noise and an increased sensitivity for mismatches in microphones characteristics [8].

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2. CONSTRUCTION OF EIGENBEAMS

We know from [7], that by using a circular array of at least three (omni- or uni-directional microphone) sensors in a planar geometry and the application of signal processing techniques, it is possible to construct a first-order superdirectional response that can be steered with its main-lobe to any desired azimuthal angle and can be adjusted to have any first-order directivity pattern (cardioid, hypercardioid, etc.). This construction is performed via so-called zeroth- and first-order eigenbeams. For wavelengths larger than the size of the array[1] and assuming that we have no sensor-noise, the responses of the eigenbeams are frequency invariant and ideally equal to:

$$E_m = 1 \quad (1)$$

$$E_d^0(\theta,\phi) = \cos(\phi)\sin(\theta) \quad (2)$$

$$E_d^{\pi/2}(\theta,\phi) = \cos(\phi-\pi/2)\sin(\theta), \quad (3)$$

with θ and ϕ the standard spherical coordinate angles: elevation and azimuth. The directivity patterns of these eigenbeams are shown in Fig. 1.

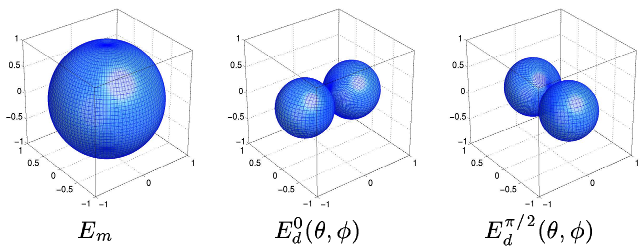

Fig. 1. Eigenbeams (monopole and two orthogonal dipoles).

[1]For wavelengths smaller than the size of the array, spatial aliasing effects will occur.

The zeroth-order eigenbeam E_m represents the monopole response, while the first-order eigenbeams $E_d^0(\theta, \phi)$ and $E_d^{\pi/2}(\theta, \phi)$ represent the orthogonal dipole responses.

The dipole can be steered to any angle φ_s by means of a weighted combination of the orthogonal dipole pair:

$$E_d^{\varphi_s}(\theta, \phi) = \cos(\varphi_s)\, E_d^0(\theta, \phi) + \sin(\varphi_s)\, E_d^{\pi/2}(\theta, \phi), \quad (4)$$

with $0 \le \varphi_s \le 2\pi$ the steering angle.

Finally, the steered and scaled superdirectional microphone response can be constructed via:

$$\begin{aligned} E(\theta, \phi) &= S\,[\alpha\, E_m + (1-\alpha)\, E_d^{\varphi_s}(\theta, \phi)] \\ &= S\,[\alpha + (1-\alpha)\cos(\phi - \varphi_s)\sin(\theta)]\,, \end{aligned} \quad (5)$$

with $\alpha \le 1$ the parameter for controlling the directional pattern of the first-order response and S an arbitrary scaling factor (which can also have negative values).

In the remainder of this paper, we will assume that we have a unity response of the superdirectional microphone for a desired source coming from an arbitrary azimuthal angle ϕ and for an elevation angle $\theta = \frac{\pi}{2}$.

3. DIFFUSE NOISE

For analyzing the microphone response in the presence of spherically isotropic diffuse noise[2], we use the directivity factor Q given by [4] [5]:

$$Q = \frac{4\pi}{\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} E^2(\theta, \phi)\, \sin\theta\, d\theta\, d\phi}. \quad (6)$$

If we combine Eq. (5) with Eq. (6) and perform the integration, we see that the directivity factor Q is expressed as:

$$Q = \frac{3}{(1 - 2\alpha + 4\alpha^2)\, S^2}. \quad (7)$$

To obtain a unity desired response at angle φ_s, we have to choose $S = 1$. The directivity index $DI = 10\log_{10}(Q)$ is plotted as function of α in Fig. 2. Here it can be seen that for $\alpha = \frac{1}{4}$ we get the maximum directivity index of 6 dB, corresponding to a first-order hypercardioid. For $\alpha = \frac{1}{2}$ and $\alpha = 1$, we get respectively the first-order cardioid and the monopole response with respectively a directivity index DI of 4.8 and 0 dB.

The maximum reduction of diffuse noise is obtained for $\alpha = \frac{1}{4}$. However, when we also want to remove a point interferer coming from some other angle, choosing $\alpha = \frac{1}{4}$ will not automatically yields the best result. The situations with a single point interferer will be considered next.

[2]Due to the limited length of this paper, we only focus on spherically isotropic diffuse noise. However, the results of this paper can be easily translated to the cylindrically isotropic diffuse noise case. Furthermore, we note that reverberant acoustic fields are often modeled as spherically isotropic diffuse noise.

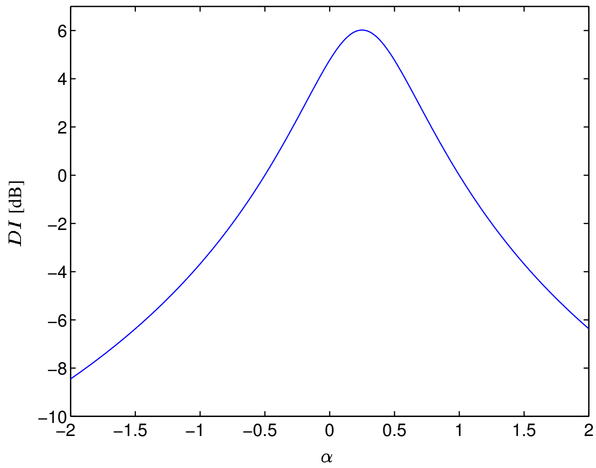

Fig. 2. Directivity index DI as function of α.

4. SINGLE POINT INTERFERER

The first-order superdirectional response (with a unity response at angle φ_s) has maximally two nulls on the azimuthal plane. The angles φ_n of these two nulls can be found by solving $|E(\theta, \phi)|^2 = 0$ with $\theta = \frac{\pi}{2}$ and are given by [5]:

$$\varphi_n = \varphi_s \pm \arccos\left(\frac{-\alpha}{1-\alpha}\right), \quad (8)$$

where a single null is obtained for $\alpha = \frac{1}{2}$ and two nulls are defined for $\alpha < \frac{1}{2}$.

Next, we want to construct a directional pattern in such a way that we obtain a unity response at angle φ_s under the constraint that a null is placed at angle φ_n. To enable this directional pattern, we use Eq. (8) to obtain the value of α which is given by [4]:

$$\alpha = \frac{\cos(\varphi_n - \varphi_s)}{\cos(\varphi_n - \varphi_s) - 1}. \quad (9)$$

From Eq. (9), it can be seen that the value of α becomes more negative when the angular difference between φ_n and φ_s is smaller. In the limiting case where φ_n is equal to φ_s, we obtain $\alpha = -\infty$.

For this simple null-steering scheme (hereafter called the baseline method), it is also useful to look at the directivity factor for spherically diffuse noise, as given in Eq. (6). The directivity factor be computed by combining Eq. (9) with Eq. (7). In this way, we get:

$$Q = \frac{3\,[1 - \cos(\varphi_n - \varphi_s)]^2}{1 + 3\cos^2(\varphi_n - \varphi_s)}. \quad (10)$$

Fig 3 shows the directivity index DI as function of $\varphi_n - \varphi_s$, where it can be seen that for $\varphi_n - \varphi_s = \pm 1.91$ rad. a hypercardioid is obtained with a directivity index of 6 dB.

Furthermore, for the limit-case of $\varphi_n = \varphi_s$, the directivity index goes to $-\infty$ dB. For $\varphi_n - \varphi_s = \pm\pi$ rad. a cardioid response is obtained with a directivity index of 4.8 dB.

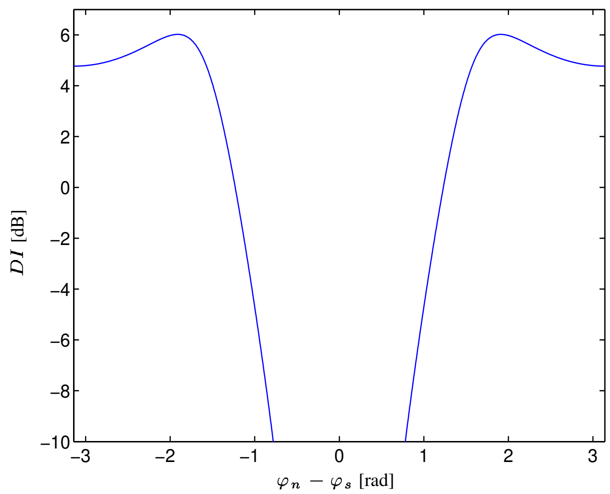

Fig. 3. Directivity index DI as function of $\varphi_n - \varphi_s$.

As the baseline method does not have a very good directivity index for $|\varphi_n - \varphi_s| < \frac{\pi}{2}$, we propose an alternative method as discussed hereafter.

5. POINT INTERFERER WITH DIFFUSE NOISE

To obtain the best possible directivity factor Q under the constraints that we obtain a unity response at an angle (hereafter defined as $\tilde{\varphi}_s$) and a null is obtained at another angle φ_n, it is generally not optimal (in terms of maximizing the directivity factor Q) to steer the main-lobe (with unity response) to $\tilde{\varphi}_s$ (as was done for the baseline method). Therefore, hereafter, we will use the angle φ_s and scale-factor S as extra degrees of freedom to optimize the directivity factor under the constraint that a unity response is obtained at angle $\tilde{\varphi}_s$.

Via Eq. (5), we can see that a unity desired response at angle $\tilde{\varphi}_s$ is obtained when we choose the scale-factor S as:

$$S = \frac{1}{\alpha + (1-\alpha)\cos(\tilde{\varphi}_s - \varphi_s)}. \qquad (11)$$

In the same way as in the previous section, we can choose α in such a way that a null is obtained at angle φ_n:

$$\alpha = \frac{\cos(\varphi_n - \varphi_s)}{\cos(\varphi_n - \varphi_s) - 1}. \qquad (12)$$

Combining Eq. (12) and Eq. (11) with Eq. (7) yields:

$$Q = \frac{3\left[\cos(\tilde{\varphi}_s - \varphi_s) - \cos(\varphi_n - \varphi_s)\right]^2}{1 + 3\cos^2(\varphi_n - \varphi_s)}. \qquad (13)$$

We can compute the extrema of Eq. (13) by taking the derivative with respect to φ_s and setting the resulting derivative to zero. The maximum directivity factor Q is obtained for two solutions of φ_s:

$$\varphi_s = \varphi_n - 2\arctan\left[\frac{1 - \cos(\varphi_n - \tilde{\varphi}_s) \pm \sqrt{A}}{4\sin(\varphi_n - \tilde{\varphi}_s)}\right], \qquad (14)$$

with:

$$\begin{aligned} A &= \cos^2(\tilde{\varphi}_s - \varphi_n) + 16\sin^2(\tilde{\varphi}_s + \varphi_n) - 2\cos(\tilde{\varphi}_s - \varphi_n) \\ &\quad +1 - 64\cos(\tilde{\varphi}_s)\cos(\varphi_n)\sin(\tilde{\varphi}_s)\sin(\varphi_n). \qquad (15) \end{aligned}$$

Although these two solutions of φ_s are different, and lead to different values of α and S, the directional responses are exactly the same.

For the newly proposed method, the directivity index DI is shown as function of $\varphi_n - \tilde{\varphi}_s$ in Fig. 4. Comparing this plot with Fig. 3 of the baseline method, it can be seen that the newly proposed method always leads to equal or better results for any value of $\varphi_n - \tilde{\varphi}_s$. Especially when $|\varphi_n - \tilde{\varphi}_s| < \frac{\pi}{2}$, the newly proposed method has a much better directivity index compared to the baseline method.

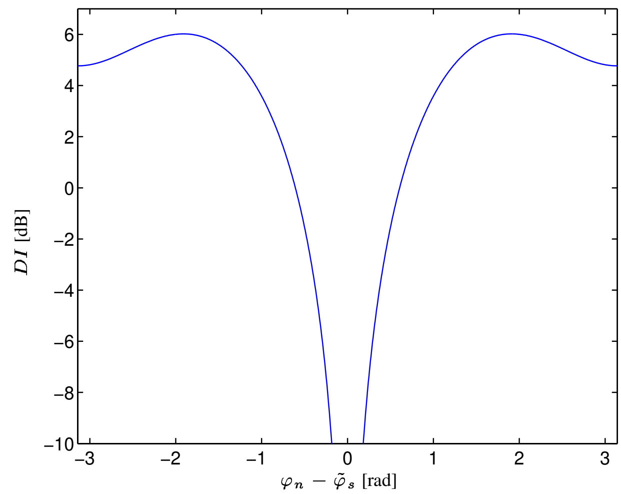

Fig. 4. Directivity index DI as function of $\varphi_n - \tilde{\varphi}_s$.

It is interesting to note that just as for the baseline method (shown in Fig. 3) for $\varphi_n - \tilde{\varphi}_s = \pm 1.91$ rad. the maximum is obtained at 6 dB (hypercardioid) and for $\varphi_n - \tilde{\varphi}_s = \pm\pi$ rad. the maximum is obtained at 4.8 dB (cardioid).

6. VALIDATION

To compare the newly proposed steering method, discussed in Section 5 with the baseline method, discussed in Section 4, we constructed the directional responses for both cases when obtaining a unity desired response toward 90 degrees and preserving a null at respectively the angles -30, 0, 30 and 45 degrees.

For the baseline method, we obtain the values as shown (in bold) in Table 1. For this case, the steering angle φ_s is always equal to the angle where we want to obtain a unity desired response. Furthermore, the scale factor S always equals unity. When φ_n becomes closer to the steering angle φ_s, the value of α decreases and also the directivity index becomes worse.

For the newly proposed method, we obtain the values as

φ_n [deg]	φ_s [deg]	S	α	DI
-30	90	1	**0.333**	5.86
0	90	1	**0**	4.78
30	90	1	**-1**	-3.68
45	90	1	**-2.372**	-9.88

Table 1. Computed values for the baseline method.

φ_n [deg]	$\tilde{\varphi}_s$ [deg]	φ_s [deg]	α	S	DI
-30	90	**83.4**	**0.284**	**1.005**	5.95
0	90	**104.0**	**0.195**	**1.025**	5.74
30	90	**128.2**	**0.125**	**1.231**	3.86
45	90	**140.9**	**0.093**	**1.504**	1.94

Table 2. Computed values for the newly proposed method.

shown (in bold) in Table 2. In contrast to the baseline method, the angle φ_s is different compared to the angle where we want to obtain a unity desired response. Furthermore, it can be seen that $\alpha > 0$ for all cases, leading to improved values of the directivity index DI. The polar-plots for the baseline method and the newly proposed method are shown in Fig. 5.

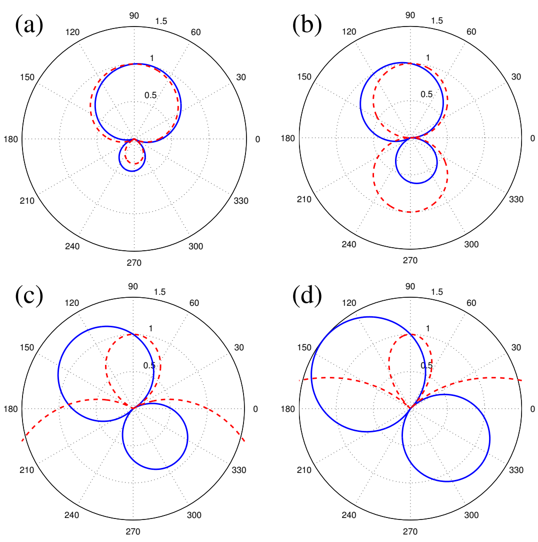

Fig. 5. Azimuthal polar-plots for the baseline method (dashed) and the new method (solid) will nulls placed at **(a)** -30 degrees, **(b)** 0 degrees, **(c)** 30 degrees and **(d)** 45 degrees.

From the plots in Fig. 5, it is clear that when the angle between the desired source and the interfering source is small (e.g. in Fig. 5d), the newly proposed method has a much better directivity index. For the newly proposed method, the directivity pattern generally has an asymmetric behaviour around the desired angle $\tilde{\varphi}_s$. For the baseline method, a symmetric behaviour around the desired angle φ_s is visible, but at the cost of a large maximum response at angle $\varphi_s \pm \pi$ in case $\alpha < 0$, resulting in degradation of directivity index (and also amplification of sensor-noise). Via Eq. (5), it can be derived that this maximum response equals $S(1 - 2\alpha)$ for $\alpha < 0$.

7. CONCLUSIONS

In this paper, a new technique is presented that enables the steering of a first-order superdirectional microphone response toward a desired angle to capture a target signal, while preserving a null located at a predefined angle to suppress an interfering signal. The main benefit of the new technique (over standard null-steering schemes) is that the response for (spherical isotropic) diffuse-noise is also minimized. Especially when the angle between the target signal and the interferer signal is small, the new technique is superior compared to the standard null-steering method.

8. REFERENCES

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