AN EXPERIMENTAL STUDY OF THE EIGENDECOMPOSITION METHODS FOR BLIND SIMO SYSTEM IDENTIFICATION IN THE PRESENCE OF NOISE

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ABSTRACT

Subspace methods for blind SIMO system identification have been proposed which rely on the null subspace of the data correlation matrix to estimate the impulse response coefficients. It is known that the performance of these algorithms degrades with increasing noise and large system orders. In this paper, we present results of an experimental study that links the performance of the subspace algorithm with the eigenvalues of the multichannel input correlation matrix. It is demonstrated that the eigenvector corresponding to the smallest eigenvalue is not always the best solution neither in terms of normalised projection misalignment nor cross-relation error.

1. INTRODUCTION

Blind SIMO system identification is a commonly occurring problem with several applications in various fields of engineering and in particular where blind deconvolution or source separation is required. Example areas of application include communications where the received signal must be equalised to obtain the transmitted signal, geophysics where the reflectivity of the earth layers is explored by extracting seismic signals from the sensor observations [1] and reverberant speech enhancement where the aim is to estimate the acoustic impulse responses blindly from reverberant observations, and then deconvolve to remove the effects of the room [2].

Algorithms for blind identification of SIMO systems based on the cross-relation between two channels have received a great deal of attention in the literature with various different implementations [1, 2, 3, 4, 5, 6]. However, all these algorithms in common suffer from several problems which need to be overcome before they can by applied in practice. These problems include presence of common zeros in the channel transfer functions [7, 8], unknown channel orders, difficulty in handling long impulse responses and sensitivity to additive noise [9].

In this paper, we investigate the effects of noise by monitoring the eigenvalues of the correlation matrix. This investigation focuses on the cross-relation error measure and the normalised projection misalignment, both of which are standard measures in this context and will be defined in Section 4. It is demonstrated that, in the presence of noise, both these measures may be minimised by subspace vectors that do not necessarily correspond to the smallest eigenvalue of the correlation matrix.

The remainder of the paper is organised as follows. In Section 2 the problem of blind system identification is formulated and the eigenvalue decomposition algorithm is reviewed in Section 3. The rationale and the motivation for the experiments are described in Section 4. The simulation results are presented in Section 5 followed by conclusions in Section 6.

2. PROBLEM FORMULATION

Consider a signal, $s(n)$, produced in a noisy multipath environment and observed by an array of sensors at a distance from the source. The signal received at the mth sensors can be written

$$y_m(n) = \mathbf{h}_m^T \mathbf{s}(n), \quad (1)$$
$$x_m(n) = y_m(n) + \nu_m(n), \quad (2)$$

where $\mathbf{h}_m = [h_{m,0}\ h_{m,1}\ \dots\ h_{m,L-1}]^T$ is the L-tap impulse response of the acoustic path between the source and the mth microphone, $\mathbf{s}(n) = [s(n)\ s(n-1)\ \dots\ s(n-L+1)]^T$ is an input vector of source samples and $\nu_m(n)$ is ambient noise. The aim of a blind channel identification algorithm is to form an estimate, $\hat{\mathbf{h}}_m$, of the impulse responses, $\mathbf{h}_m$, using only the noisy observations $x_m(n),\ m = 1, 2, \dots, M$. This has been shown possible if the following identifiability conditions are satisfied [3]: (i) the channels do not share any common zeros and (ii) the autocorrelation matrix of the source signal is of full rank.

3. EIGENDECOMPOSITION BLIND SIMO SYSTEM IDENTIFICATION

In this section, the eigendecomposition method for blind system identification presented in, e.g. [2] is summarised.

For the work presented in this paper we consider the simplest multichannel case, $M = 2$, for tractability. The concepts are straightforwardly extended to the general M-channel case. In a noise-free scenario, the following cross relation between the channels and the observed signals holds [3, 2]

$$\mathbf{h}_1^T \mathbf{y}_2(n) = \mathbf{h}_2^T \mathbf{y}_1(n), \tag{3}$$

where $\mathbf{y}_m(n) = [y_m(n)\ y_m(n-1)\ \ldots\ y_m(n-L+1)]^T$ is a vector of observation samples at the mth sensor.

A two-channel data matrix may be constructed as

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_2 \\ -\mathbf{Y}_1 \end{bmatrix} \tag{4}$$

and consequently, the multichannel correlation matrix can be written

$$\mathbf{R} = \frac{1}{N+1} \mathbf{Y}\mathbf{Y}^T. \tag{5}$$

Thus, in terms of this correlation matrix the following system of equations is formulated

$$\mathbf{R}\mathbf{h} = 0, \tag{6}$$

where $\mathbf{h} = [\mathbf{h}_1^T\ \mathbf{h}_2^T]^T$ is a vector with concatenated channel impulse responses.

From (6) it is seen that the channel vector is a vector in the null subspace of $\mathbf{R}$ which can be found using eigenvalue decomposition (EVD). It was shown in [4] that the rank of the null subspace is, $\hat{p}_h - p_h + 1$ where $\hat{p}_h \geq p_h$ is an estimated value of the channel order. The eigenvalues can be sorted in the following manner

$$\begin{cases} \lambda_l = 0, & l = 0, 1, \ldots, \hat{p}_h - p_h \\ \lambda_l > 0, & \text{otherwise} \end{cases} \tag{7}$$

The true order can be found from (7), provided that the null subspace is correctly identified. For the remainder of this work, we will assume that L is known. The corresponding eigenvectors of the null subspace provide the coefficients of the impulse response up to an arbitrary scaling factor. By assuming white noise conditions, the null subspace of (5) will have a value equal to the noise power rather than zero.

4. STUDY DESCRIPTION AND RATIONALE

We now describe and motivate the experimental study conducted for this paper. In the absence of noise, the algorithm is capable of estimating the channel transfer functions accurately. In contrast, previous studies have shown that the performance of the algorithm degrades with increasing noise levels [5, 2, 9]. In particular, at a lower SNR value of 20 dB, the NPM is very large and the subspace method cannot correctly identify the impulse responses. Noise may also effect the impulse response and can cause common zeroes amongst the channels, which reduces the accuracy of the algorithm. In addition, for increasing channel orders, the performance also decreases as the algorithm is sensitive to errors in the EVD and cannot find channel coefficients accurately. A better understanding of the underlying cause of such degradations would prove beneficial for improvement of the algorithms.

In this experimental study, we investigate the distribution of the eigenvalues of the correlation matrix and how this is affected by noise. We form a histogram using 10 uniform bins over the range of eigenvalues. The values obtained from the first bin of the distribution are studied since the true channels are obtained from the null subspace eigenvalues or the smallest eigenvalues.

Two standard measures of performance are considered for the experiment. First, the normalised projection misalignment (NPM), defined as [10]

$$\text{NPM} = 20 \log_{10} \left(\frac{\|\mathbf{h} - \beta\hat{\mathbf{h}}\|}{\|\mathbf{h}\|} \right) \text{dB}, \tag{8}$$

with

$$\beta = \frac{\mathbf{h}^T \hat{\mathbf{h}}}{\hat{\mathbf{h}}^T \hat{\mathbf{h}}}, \tag{9}$$

where $\hat{\mathbf{h}}$ is the estimate of the concatenated impulse responses $\mathbf{h}$. The projection of $\mathbf{h}$ onto the estimated channel, $\hat{\mathbf{h}}$, will take into account the intrinsic misalignment of the channel estimate, disregarding the arbitrary gain factor. Next, in the presence of noise, the cross-relation error between two channels is defined [5]

$$e(n) = \hat{\mathbf{h}}_1^T \mathbf{y}_2(n) - \hat{\mathbf{h}}_2^T \mathbf{y}_1(n). \tag{10}$$

The relation of (10) can be written for each pair of channels when $M > 2$. According to theory, the transfer function coefficients corresponding to the null subspace will satisfy (10), such that the mean squared error is a minimum. For each eigenvalue from the first bin of the distribution, we use the corresponding eigenvectors to calculate the cross-relation error and NPM. The expected outcome is that the lowest NPM and cross-relation error would be obtained from the eigenvectors corresponding to the smallest eigenvalue.

5. SIMULATION RESULTS

In this section, we present and discuss the results from the simulations. We compare the noise-free case with a noisy case with $SNR = 10$ dB.

In the first set of experiments a two channel system was considered with $L = 128$ taps. Figure. 1 shows the results for the noise free case in terms of (a) NPM, (b) cross-relation error and (c) the eigenvalues of the first bin. As expected, the NPM in Fig. 1a has a value indicating

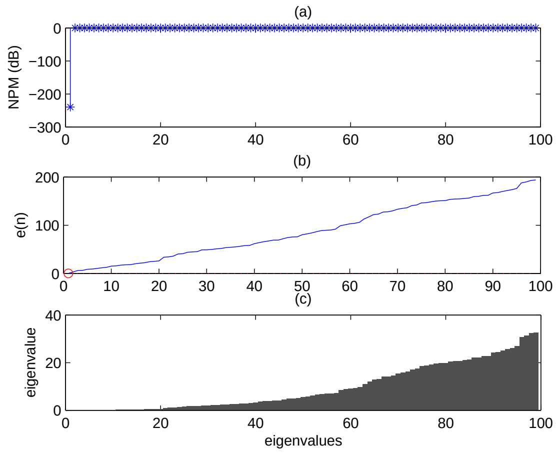

Figure 1: *Experimental results for the noise-free case (SNR* $= \infty$ *dB) with random channels of length* $L = 128$ *in terms of (a) NPM, (b) Cross-relation error* $e(n)$ *and (c) correlation matrix eigenvalues.*

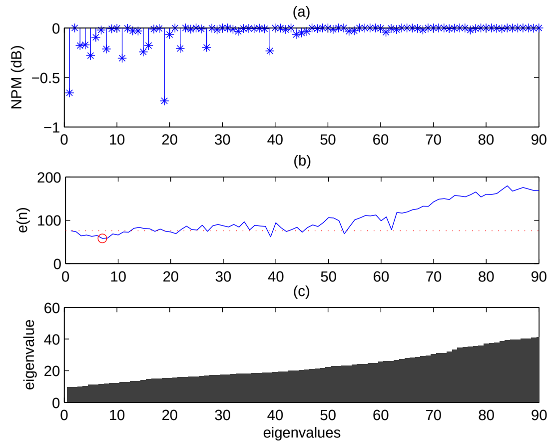

Figure 2: *Experimental results for the noisy case (SNR* $=$ *10 dB) with random channels of length* $L = 128$ *in terms of (a) NPM, (b) Cross-relation error* $e(n)$ *and (c) correlation matrix eigenvalues.*

exact identification for the first eigenvalue; whereas it is zero for other the other eigenvalues in the bin. The corresponding cross-relation error of (10) is also minimised for this eigenvalue and the error then increases linearly with increasing eigenvalues. On the contrary, for the noisy case it is shown that the NPM in Fig. 2a and the cross-relation error Fig. 2b are minimised by eigenvectors that do not correspond to the smallest eigenvalue where the error due to the smallest eignevalue is indicated with a dotted line in Fig. 2b. Furthermore, the cross-relation error and the NPM have their minima at different values.

Apart from the minimum error indicated with a circle, there are several other error values that lie below this line. This indicates that the best solution may not always come from the smallest eigenvalue. By comparing the plots of the eigenvalues in the first bin of the histogram, it is noticeable that the first fifteen eigenvalues are much smaller in the noiseless case than in the noisy case. These result can vary between experiments and we have observed some cases for which the minimum error and the minimum NPM may correspond to the smallest eigenvalue. The effect of noise may have been to introduce common zeros by shifting near-common zeros closer together. In order to avoid this effect we conducted a second set of experiments using shorter impulse responses and making sure that there are no common or near common zeros. The zeros of the two random channels of length $L = 16$ used are depicted in Fig. 3.

The results of this second experiment are presented in Figure 4 in terms of (a) NPM, (b) cross-relation error and (c) eigenvalues. Again, it is observed that the NPM and cross-relation error are not obtained with the eigenvector

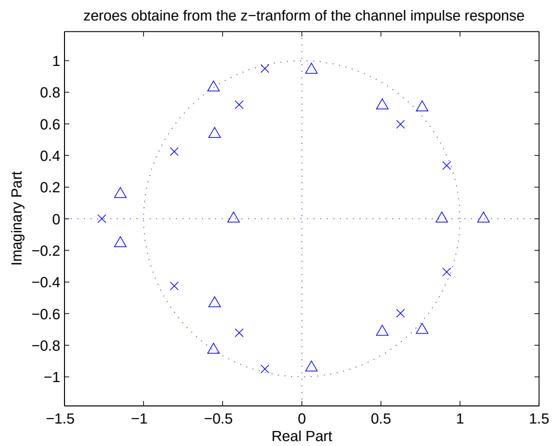

Figure 3: *Channel zeros for a two channel system with random impulse responses of length* $L = 16$.

corresponding to the smallest eigenvalue. Instead, the second eigenvector of the bin provides the lowest NPM, while the fourth eigenvector provides the smallest cross-relation error.

The results presented in this section raise interesting issues in relation to the adaptive methods of blind channel identification which are based on iterative minimisation of the cross relation error. The results of this study may be the initial steps to the explanation of the observed problems with misconvergence [9, 11] of the algorithms in the presence of noise.

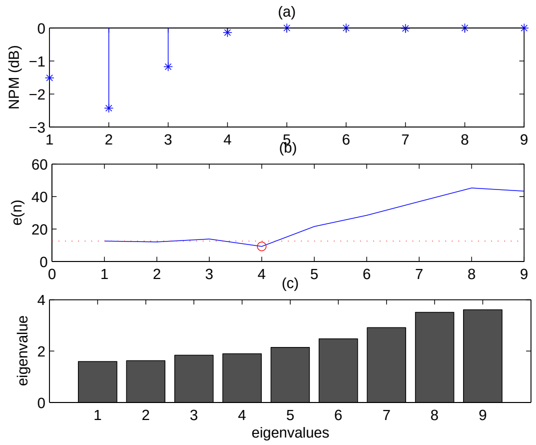

Figure 4: *Experimental results for the noisy case (SNR =* 10 *dB) with random channels of length* $L = 16$ *in terms of (a) NPM, (b) Cross-relation error* $e(n)$ *and (c) correlation matrix eigenvalues.*

6. CONCLUSION

The subspace methods for blind channel identification and its performance in the presence of noise have been discussed. An experimental study was presented where the eigenvalues of the correlation matrix of a two channel SIMO system correlation matrix were observed and compared for a noise-free and a noisy case. The distribution of the eigenvalues was considered and those in the first bin of a histogram were chosen for experimental study. Consequently, the normalised projection misalignment and the cross-relation error were calculated for each eigenvector corresponding to an eigenvalue of the first bin. It was demonstrated that the smallest eigenvalue does not necessarily minimise the NPM nor the cross-relation error. This experimental study therefore provides some insight into the noise robustness of the subspace-based approach.

This encourages future work on this topic which would involve a more detailed study of the effect of noise on the roots of the system transfer function and enhancement of the subspace algorithm to determine a better estimate of the coefficients.

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