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A Theoretical Analysis for Feedforward ANC System with State Equation Model

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Abstract

In this paper, a system architecture and its adaptive algorithm for feedforward ANC system are presented. The proposed system architecture consists of a multi-input / multi-output adaptive linear combiner and secondary source propagation space which is described by state equation model. Some theoretical discussions on equivalent system, cost function and optimal weight parameters are provided. Also an adaptive algorithm is derived based on steepest-descent method. Finally, by a numerical simulation example for noise attenuation in closed room, it can be found that the proposed architecture and algorithm work effectively.

1. Introduction

For multi-input / multi-output control system, it is considerable to use state equation description of modern control system theory, because it is very convenience to cope with the difficulty different number of input and output, and also many theoretical property can be used. We have proposed a system architecture and an adaptive algorithm for feedforward ANC system with state equation model, it achieved very good performance for noise attenuation in a closed room [1]. However, these previous works are made for practical case. In this paper, we provide some theoretical discussions on equivalent system, cost function and optimal weight parameters. Our research has two points. The first one is for theoretical means. We show that a feedforward ANC system can be replaced by a preconditional adaptive filter, it means that we can provide optimal weight parameters analytically and an algorithm of updating weight parameter in a matrix form like Filtered-X LMS algorithm. The second point is for practical means. Although a FIR model for secondary source propagation space has been presented [2], the proposed architecture can include both of FIR and IIR model, however, IIR model is able to be used for describing acoustical pole characteristics in room.

2. System Description with State Equation Model

For giving a formulation of our problem, we describe the feedforward ANC system with two parts as following:

(1) a multi-input / multi-output adaptive controller, which has an adjustable weight parameter matrix W.

(2) a sound propagation space for secondary source which is described by a state equation model.

The system block diagram is illustrated in Fig.1.

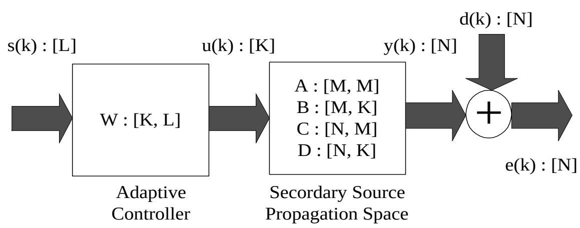

Fig.1 Block diagram of feedforward ANC system

The adaptive controller is implemented by a multi-input / multi-output adaptive linear combiner as shown in Fig.2.

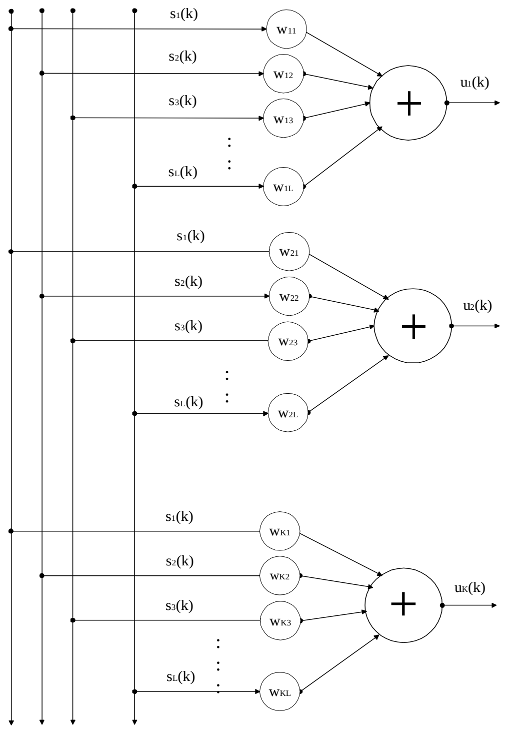

Fig.2 Diagram of multi-input / multi-output adaptive linear combiner

The secondary source vector u(k) is given by combining weighted input vector s(k) as shown in following equation.

$$u(k) = Ws(k) \tag{1}$$

where $W = [w_{ij}]$ is a $K \times L$ weight parameter matrix.

The sound propagation space for secondary source is described by state equation as following,

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ y(k) &= Cx(k) + Du(k) \\ x(0) &= 0_M \end{aligned} \tag{2}$$

where x(k) is a M degree state vector and y(k) is a N degree output vector and A, B, C, D are coefficient matrix of state equation. When a noise vector d(k) is added, the ANC system makes cancellation by output vector y(k), and the residual error noise vector e(k) will be detected by microphones, it is given by

$$e(k) = d(k) - y(k) \tag{3}$$

Our problem is to minimize a cost function related to this residual error noise vector e(k) by adjusting weight parameters.

3. Equivalent System

Here we will give an equivalent system for feedforward ANC system shown in Fig.1. If we denote that

$$H_0 = D \tag{4}$$

$$H_l = CA^{l-1}B \text{, where } l = 1,2,3...k \tag{5}$$

we can give the output vector y(k) of state equation as following.

$$y(k) = [y_{1,} y_{2,} \cdots y_N]^T = \sum_{l=0}^{k} H_l u(k-l) \tag{6}$$

Furthermore, by denote $H_l = [h_{lij}]$, the *i*-th element of y(k) can be written as following.

$$\begin{aligned} y_i(k) &= \sum_{l=0}^{k}\sum_{j=1}^{K} h_{lij} u_j(k-l) = \sum_{l=0}^{k}\sum_{j=1}^{K} h_{lij} \sum_{m=1}^{L} w_{jm} s_m(k-l) \\ &= \sum_{l=0}^{k}\sum_{j=1}^{K}\sum_{m=1}^{L} h_{lij} w_{jm} s_m(k-l) = \sum_{j=1}^{K}\sum_{m=1}^{L} w_{jm} \sum_{l=0}^{k} h_{lij} s_m(k-l) \end{aligned} \tag{7}$$

Also if we denote that

$$z_{mij}(k) = \sum_{l=0}^{k} h_{lij} s_m(k-l) \tag{8}$$

then Eq.(7) can be rewritten as following,

$$y_i(k) = \sum_{j=1}^{K}\sum_{m=1}^{L} w_{jm} z_{mij}(k) \tag{9}$$

So that the output vector y(k) is denoted in matrix form as following.

$$y(k) = W \bullet \hat{Z}(k) \tag{10}$$

where $\hat{Z}(k)$ is a $L \times N \times K$ matrix with elements of $z_{mij}(k)$. The operation of $\bullet$ is defined by Eq.(9). This equation shows that output vector y(k) can be given by a sum of production of the weight parameter matrix of adaptive controller and a output matrix from an alternate system for second source propagation space. This system can be considered as a preconditional adaptive filter with KL weight parameters. The block diagram of this equivalent system is shown in Fig.3.

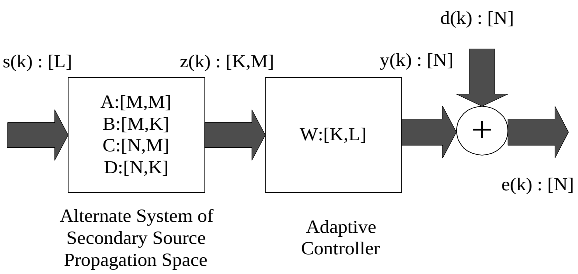

Fig.3 Equivalent block diagram of Fig.1

The alternate system of secondary source propagation space can be given by following extanded state equation.

$$\begin{aligned} \hat{V}(k+1) &= A\otimes\hat{V}(k)+B\otimes\hat{S}(k) \\ \hat{Z}(k) &= C\otimes\hat{V}(k)+D\otimes\hat{S}(k) \\ \hat{V}(0) &= 0_{L\times M\times K} \end{aligned} \tag{11}$$

where $\hat{S}(k)$ is a $L\times K\times K$ matrix with layer of

$$S_m(k)=\begin{bmatrix} s_m(k) & 0 & \cdots & 0 \\ 0 & s_m(k) & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \cdots & s_m(k) \end{bmatrix} \tag{12}$$

and $\hat{V}(k)$ is a $L\times M\times K$ matrix with layer of

$$V_m(k)=\begin{bmatrix} v_{m11} & v_{m12} & \cdots & v_{m1K} \\ v_{m21} & v_{m22} & \cdots & v_{m2K} \\ \vdots & \vdots & \ddots & \vdots \\ v_{mM1} & v_{mM2} & \cdots & v_{mMK} \end{bmatrix} \tag{13}$$

and $\hat{Z}(k)$ is a $L\times N\times K$ matrix with layer of

$$Z_m(k)=\begin{bmatrix} z_{m11} & z_{m12} & \cdots & z_{m1K} \\ z_{m21} & z_{m22} & \cdots & z_{m2K} \\ \vdots & \vdots & \ddots & \vdots \\ z_{mN1} & z_{mN2} & \cdots & z_{mNK} \end{bmatrix} \tag{14}$$

also, $m=1,2,3\cdots L$ in Eq.(12), Eq.(13) and Eq.(14). For a 2-dimensional matrix P and 3-dimensional matrix $\hat{Q}$, the operation of $\otimes$ is defined by following equation.

$$\hat{G}=P\otimes\hat{Q}=\left[g_{mij}\right]=\left[\sum_{f=1}^{F}p_{if}\,q_{mfj}\right] \tag{15}$$

4. Optimal Solution

The cost function to be minimized can be defined as the sum of the mean-square error signal as following.

$$J=E\left[\sum_{m=1}^{N}e_m^2(k)\right] \tag{16}$$

From Fig.3, the multi-input / multi-output ANC problem is replaced to a preconditional adaptive digital filter problem, so we can use Wiener solution to show optimal weight as following.

$$cs(W_{opt})=R_{\hat{Z}\hat{Z}}^{-1}cs(R_{\hat{Z}d}) \tag{17}$$

where $R_{\hat{Z}\hat{Z}}=\left[\sum_{m=1}^{N}E(z_{lmj}z_{nmi})\right]$ is a auto-correlation matrix of $\hat{Z}(k)$, $R_{\hat{Z}d}=\left[\sum_{m=1}^{N}E(d_m z_{lmj})\right]$ is a cross-correlation matrix of $\hat{Z}(k)$ and $d(k)$, cs(A) means column string of matrix A.

5. Adaptive Algorithm

Based on the steepest-descent method, in order to make the cost function reach its minimum value, the updating formulation of the weight parameter matrix is expressed by following formulation.

$$W(k+1)=W(k)-\frac{1}{2}\mu\nabla\xi \tag{18}$$

where ξ is transient estimation of cost function in Eq.(16), which is given by following equation.

$$\xi=tr[e(k)e^T(k)]=\sum_{m=1}^{N}e_m^2(k) \tag{19}$$

and $\nabla\xi$ denotes the gradient of ξ, which is given as following.

$$\nabla\xi=\frac{\partial\xi}{\partial W}=-2\sum_{m=1}^{N}e_m(k)\frac{\partial y_m(k)}{\partial W} \tag{20}$$

Because the *ij*-th element of $\dfrac{\partial y_m(k)}{\partial W}$ is given as following,

$$\frac{\partial y_m(k)}{\partial w_{ij}} = \frac{\partial}{\partial w_{ij}}(\cdots + w_{ij} z_{mij} + \cdots) = z_{mij}(k) \qquad (21)$$

the update equation of weight parameters can be shown as following.

$$w_{ij}(k+1) = w_{ij}(k) - \mu \sum_{m=1}^{N} e_m(k) z_{mij}(k) \qquad (22)$$

Here by denoting an operation of $\oplus$ between a 1-dimensional matrix (i.e. vector) and a 3-dimensional matrix as,

$$e(k) \oplus \hat{Z}(k) = \left[\sum_{m=1}^{N} e_m(k) z_{mij}(k) \right] \qquad (23)$$

the updating formulation of weight parameters in matrix form can be shown as following. This is a extended form of Filtered-X LMS algorithm.

$$W(k+1) = W(k) + \mu e(k) \oplus \hat{Z}(k) \qquad (24)$$

6. Numerical Simulation

A numerical simulation has made for ensuring our proposed system. In order to give description for reverberation characteristics shown as in Fig.4, we choose a 4-input and 5-output state equation with coefficient matrix in Eq.(25).

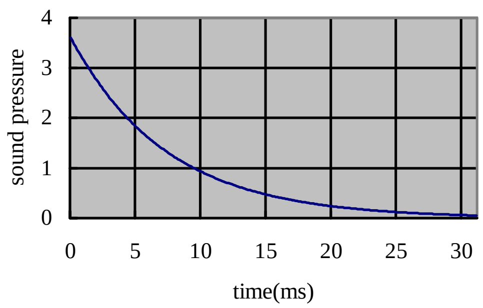

Fig.4 Reverberation characteristics of the room

$$A = \begin{bmatrix} 0.9874 & 0 & 0 & 0 \\ 0 & 0.9874 & 0 & 0 \\ 0 & 0 & 0.9874 & 0 \\ 0 & 0 & 0 & 0.9874 \end{bmatrix} \quad B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (25)$$

The simulation result is shown in Fig.5.

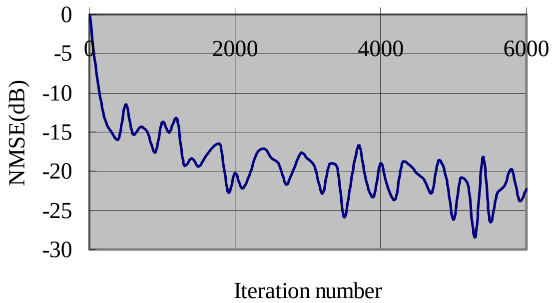

Fig.5 Convergence characteristics of proposed method

7. Conclusions

A system architecture for feedforward ANC system has been presented. This system was implemented by adaptive linear combiner and secondary source propagation space described by a multi-input / multi-output state equation. By using state equation model, we gave some discussions on equivalent system, cost function, and optimal weight parameters. Also we provided an algorithm formulation for updating weight parameters. Finally a simulation example was shown, and it shows that our system works effectively.

References

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[2] S. J. Elliott and T. J. Sutton, 1996, "Performance of feedforward and feedback systems for active control," *IEEE Trans. On Speech and Audio Processing*, vol.4, no.3, pp214-223.