Multifrequency Self-optimizing Narrowband Interference Canceller

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Abstract—The problem of cancellation of a nonstationary sinusoidal interference, acting at the output of a linear stable plant, is considered. It is assumed that disturbance is a multifrequency narrowband signal, and that system output is contaminated with wideband noise. It is not assumed that the reference signal is available. Two disturbance canceling schemes are proposed, one for disturbances with unrelated frequency components, and the other one - for disturbances with interrelated components. Simulation experiments confirm good tracking properties of the proposed control algorithms and their robustness to modeling errors.

I. INTRODUCTION

Consider the problem of cancellation of a nonstationary narrowband disturbance \( d(t) \) acting at the output of unknown complex-valued linear stable single-input single-output system (e.g. secondary acoustic path) governed by (see Fig. 1)

\[
y(t) = K_p(q^{-1})u(t-1) + d(t) + v(t)
\]  

(1)

where \( t = \ldots, -1, 0, 1, \ldots \) denotes discrete time, \( q^{-1} \) is the backward-shift operator \( q^{-1}u(t) = u(t-1) \), \( y(t) \) is the system output, \( u(t) \) is the control (cancellation) signal, \( v(t) \) is a wideband noise and \( K_p(q^{-1}) \) denotes unknown transfer function of the controlled plant.

The disturbance \( d(t) \) is a sum of \( m \) nonstationary complex sinusoidal signals (cisoids)

\[
d(t) = \sum_{i=1}^{m} d_i(t), \quad d_i(t) = \gamma_i(t)e^{j\omega_i(t)}
\]

\[
\gamma_i(t) = a_i(t)e^{j\varphi_i}, \quad \phi_i(t) = \sum_{n=1}^{t-1} \omega_i(n)
\]

(2)

where \( \omega_i(t) \) denotes a slowly time-varying instantaneous frequency and \( a_i(t) \) is a slowly time-varying (real-valued) amplitude. Note that \( \gamma_i(t) \) incorporates initial phase \( \varphi_i \) of the \( i \)-th cisoid.

The scope of applications covered by (1)–(2) is very wide and includes, among others, track following in magnetic/optical drives [1], active rejection of vibrations caused by rotating unbalanced mass [2], and cancellation of acoustic noises generated by fans or combustion engines [3].

The problem of elimination of narrowband interferences was solved by many authors, under different assumptions and using different approaches. The best-known solutions include different variants of filtered-x least mean squares (FX-LMS) algorithm [3], adaptive regulators based on internal model principle [4], or controllers based on concepts borrowed from phase-locked loops [6].

Recently a new approach, based on coefficient fixing and adaptive gain scheduling, was introduced in a series of papers [8], [9], [10]. The new schemes, called SONIC (self-optimizing narrowband interference canceller) and xSONIC (extended, frequency-adaptive version of the basic algorithm), have some unique advantages compared to existing solutions. Unlike numerous previous attempts, they require little or no prior knowledge of the plant. Furthermore, due to parsimonious controller parameterization, introduction of additional dithering signals into the loop is not necessary. Finally, they compare favorably with state-of-the-art FX-LMS regulators, both in terms of performance and computational burden.

The paper presents two multifrequency versions of SONIC, designed for unrelated and interrelated frequencies, respectively.

II. UNRELATED FREQUENCIES

Suppose that the instantaneous frequencies \( \omega_1(t), \ldots, \omega_m(t) \) are not mutually related, which usually happens when the corresponding disturbance components originate from independent sources. The multifrequency SONIC algorithm suggested in [8], [9] (but not listed there explicitly) is made up of \( m \) single-frequency SONIC controllers working in parallel and driven by the same output signal. Each subalgorith consists of three loops.
The inner loop, which computes the \( i \)-th component of the control (compensating) signal, takes the form
\[
\tilde{d}_i(t + 1|t) = e^{j\hat{\omega}_i(t)}[\tilde{d}_i(t|t - 1) + \tilde{\mu}_i(t)y(t)]
\]
\[
u_i(t) = -\frac{\tilde{d}_i(t + 1|t)}{k_n[\hat{\omega}_i(t)]}
\]  
(3)

where \( \hat{\omega}_i(t) \) denotes the estimated value of the instantaneous frequency \( \omega_i(t) \), \( \tilde{d}(t + 1|t) \) is the one-step-ahead prediction of the \( i \)-th component of the disturbance, and \( k_n[\hat{\omega}_i(t)] = K_n(e^{-j\hat{\omega}_i(t)}) \) is the nominal (assumed) steady-state gain of the plant at frequency \( \hat{\omega}_i(t) \), usually different from the true plant’s gain \( K_p(e^{-j\omega_i(t)}) \) when no prior knowledge of the plant is available, one can fix the gain by setting \( K_n(q^{-1}) = k_n \). Finally \( \tilde{\mu}_i(t) \) is a complex-valued adaptation gain. Apart from usual adjustment of the controller bandwidth to the rate of nonstationarity of the disturbance, the latter feature allows one to compensate any discrepancy between the plant \( K_p(q^{-1}) \) and its nominal model \( K_n(q^{-1}) \).

The second, intermediate loop, adjusts \( \tilde{\mu}_i(t) \) so as to minimize the local performance criterion \( V(t) = \sum_{\tau=0}^{\infty} \rho^\tau |y(t - \tau)|^2 \), where \( \rho \approx 0 \), \( 0 < \rho < 1 \), is the forgetting constant which determines the effective tuning memory length. The algorithm takes the form
\[
z_i(t) = e^{j\hat{\omega}_i(t)}[(1 - c_\mu)z_i(t - 1) - \frac{c_\mu}{\hat{\mu}_i(t - 1)}y(t - 1)]
\]
\[
r_i(t) = pr_i(t - 1) + |z_i(t)|^2
\]
\[
\tilde{\mu}_i(t) = \tilde{\mu}_i(t - 1) - \frac{y(t)z_i(t)}{r_i(t)}
\]  
(4)

where \( z_i(t) = \partial y(t, \hat{\mu}_i(t - 1))/\partial \mu_i \) denotes the output sensitivity derivative and \( c_\mu \) is a small positive constant.

Finally, the third, outer loop, provides estimates of the instantaneous frequency \( \omega_i(t) \). A simple gradient update is used
\[
g_i(t) = \text{Im} \left[ \frac{\tilde{\mu}_i(t)y(t)}{\tilde{d}_i(t|t - 1)} \right]
\]
\[
\hat{\omega}_i(t + 1) = \hat{\omega}_i(t) + \eta g_i(t)
\]  
(5)

where \( \eta > 0 \) denotes a small adaptation gain determining the speed (and accuracy) of frequency tracking.

Combining (3), (4) and (5) into one, parallel structure, one obtains the following control algorithm (see Fig. 2)
\[
z_i(t) = e^{j\hat{\omega}_i(t)}[(1 - c_\mu)z_i(t - 1) - \frac{c_\mu}{\hat{\mu}_i(t - 1)}y(t - 1)]
\]
\[
r_i(t) = pr_i(t - 1) + |z_i(t)|^2
\]
\[
\tilde{\mu}_i(t) = \tilde{\mu}_i(t - 1) - \frac{y(t)z_i(t)}{r_i(t)}
\]
\[
\tilde{d}_i(t + 1|t) = e^{j\hat{\omega}_i(t)}[\tilde{d}_i(t|t - 1) + \tilde{\mu}_i(t)y(t)]
\]
\[
u_i(t) = -\frac{\tilde{d}_i(t + 1|t)}{k_n[\hat{\omega}_i(t)]}
\]
\[
g_i(t) = \text{Im} \left[ \frac{\tilde{\mu}_i(t)y(t)}{\tilde{d}_i(t|t - 1)} \right]
\]
\[
\hat{\omega}_i(t + 1) = \hat{\omega}_i(t) + \eta g_i(t)
\]  
(6)

III. INTERRELATED FREQUENCIES

In the scheme described in the preceding section each subalgorithm takes care of one sinusoidal component of the disturbance. Since different components were assumed to be mutually unrelated, their frequencies are tracked independently of each other.

Suppose now that the estimated frequencies are mutually coupled, namely that they are multiples of the same fundamental frequency \( \omega(t) \):
\[
\omega_i(t) = l_i\omega(t), \quad i = 1, \ldots, m.
\]  
(7)

Such multiple frequencies, called harmonics, appear in the Fourier series expansions of periodic signals \( \omega(t) \equiv \omega_0 \). The use of “time-varying harmonics” can be regarded a natural extension of the Fourier analysis concepts to quasi-periodically varying processes, such as (2). The choice of multipliers \( l_i, i = 1, \ldots, m \) depends on our prior knowledge of the disturbance. When all harmonics are expected to be present, one should set \( l_i = i \). In the presence of odd harmonics only, the natural choice is \( l_i = 2i - 1 \), etc.

Suppose that the algorithm (6) is initialized in such a way that the \( i \)-th subalgorithm tracks the \( i \)-th frequency component. Note that the corresponding frequency update \( \hat{\omega}[i](t + 1) = \hat{\omega}[i](t) + \eta g_i(t) \) can be rewritten in the form \( l_i\hat{\omega}[i](t + 1) = l_i\hat{\omega}[i](t) + \eta g_i(t) \) or equivalently
\[
\hat{\omega}[i](t + 1) = \hat{\omega}[i](t) + \eta \frac{g_i(t)}{l_i}
\]

where \( \hat{\omega}[i](t) \) denotes the estimate of the fundamental frequency \( \omega(t) \) yielded by the \( i \)-th subalgorithm. The simplest way of combining such partial estimates into one “overall” estimate \( \hat{\omega}(t) \) is by means of averaging
\[
\hat{\omega}(t) = \frac{1}{m} \sum_{i=1}^{m} \hat{\omega}[i](t)
\]
Figure 3. Block diagram of feedback controller for interrelated frequencies.

which can be also expressed in a recursive form

$$\hat{\omega}(t + 1) = \hat{\omega}(t) + \eta g(t)$$

where \(g(t) = 1/m \sum_{i=1}^{m} g_i(t)/l_i\).

The multifrequency SONIC algorithm with collaborative estimation of fundamental frequency can be written down as follows (see Fig. 3)

$$z_i(t) = e^{j \hat{\omega}(t) t} (1 - c \mu) z_i(t - 1) - \frac{c \mu}{\hat{\mu}_i(t - 1)} y(t - 1)$$
$$r_i(t) = \rho r_i(t - 1) + |z_i(t)|^2$$
$$\hat{\mu}_i(t) = \hat{\mu}_i(t - 1) - \frac{y(t) z_i^*(t)}{r_i(t)}$$
$$d_i(t + 1) = e^{j \hat{\omega}(t) t} (\hat{d}_i(t) t - 1) + \hat{\mu}_i(t) y(t)$$
$$u_i(t) = \frac{d_i(t + 1) t}{\hat{k}_i l_i \hat{\omega}(t)}$$
$$g_i(t) = \text{Im} \left[ \frac{\hat{\mu}_i(t) y(t)}{d_i(t) t - 1} \right]$$
$$i = 1, \ldots, m$$
$$g(t) = \frac{1}{m} \sum_{i=1}^{m} g_i(t)$$
$$\hat{\omega}(t + 1) = \hat{\omega}(t) + \eta g(t)$$
$$u(t) = \sum_{i=1}^{m} u_i(t)$$

(8)

Remark

Although interrelated frequencies can still be estimated using the general purpose algorithm (6), incorporation of the constraint (7), whenever applicable, can significantly increase accuracy of the frequency estimates. The analysis, carried out for stationary harmonic signals by Nehorai and Porat [11], shows that significant improvements in the Cramér-Rao bounds can be achieved if the harmonic structure, imposed by (7), is taken into account in the estimation process.

On the negative side, the frequency estimates based on the harmonic model (7), may occasionally suffer from the frequency mismatch effect – \(\hat{\omega}(t)\) may lock onto a fraction or multiple of the true frequency, e.g., onto \(\frac{1}{2} \omega(t)\) or \(2 \omega(t)\) [12]. Since this effect results in attempts to track the whole bunch of nonexistent harmonics (in the case of the fractional-type convergence) or it results in the failure to track a certain number of the existing ones (in the multiple-type convergence case), it is potentially more damaging than the analogous effect that may occur in the unconstrained algorithm, which follows each frequency separately.

IV. SIMULATION RESULTS

Both frequency estimation schemes were compared with each other in a simulation experiment which featured interrelated frequencies. The impulse response of simulated secondary path, assumed to be unknown, was established in a real-world acoustic experiment. It is shown in Fig. 4.

The disturbance signal, depicted in Fig. 5, consisted of two narrowband components with instantaneous frequencies and amplitudes governed by

$$\omega_1(t) = 0.4 \pi + 0.01 \pi \sin(t/3000)$$
$$\omega_2(t) = 2 \omega_1(t)$$
$$a_1(t) = 5 + \cos(t/10000)$$
$$a_2(t) = 5 + 0.5 \sin(t/10000) .$$

Assuming 1 kHz sampling, the frequencies of both components varied in the intervals [195, 205] Hz and [390, 410] Hz. The variance of wideband background noise was equal to \(\sigma^2 = 0.01\), i.e., approximately 40 dB below the level of harmonic interference.

Both versions of xSONIC shared identical settings: \(c_\mu = 0.01, \rho = 0.999, \eta = 0.025, d_1(0) = d_2(0) = 0.01\). The nominal models were static and adjusted rather coarsely, only to guarantee initial convergence: \(k_{n,1} = -j, k_{n,2} = -1\).
Additionally, to avoid erratic behavior in the initial phase of control, the gains \( \hat{\mu}_1(t) \) and \( \hat{\mu}_2(t) \) were set to 0.05 and not updated until \( t = 2000 \). The initial conditions for the self-optimization loop were set as follows: \( z_1(2000) = z_2(2000) = 0 \), \( r_1(2000) = r_2(2000) = 100 \).

Fig. 6 shows convergence of the SONIC controller with independent frequency estimation. Somewhat poor initial behavior may be attributed to the oscillations of the frequency estimate of the second harmonic (Fig. 7). The steady-state mean-squared values of frequency estimation errors \( \Delta \hat{\omega}_i(t) = \omega_i(t) - \hat{\omega}_i(t) \), \( i = 1,2 \), and excess output error \( \Delta y(t) = y(t) - y_i(t) \) all evaluated using 30000 signal samples, were equal to \( E[|\Delta \hat{\omega}_1(t)|^2] = 1.48 \cdot 10^{-7} \), \( E[|\Delta \hat{\omega}_2(t)|^2] = 3.92 \cdot 10^{-7} \) and \( E[|\Delta y(t)|^2] = 5.51 \cdot 10^{-6} \), respectively.

The initial fragment of the output signal obtained for the controller (8) is depicted in Fig. 8. The scheme benefited from the lack of oscillations of frequency estimate in the initial control phase. However, the steady-state performance of the system has deteriorated – the corresponding estimation errors were equal to \( E[|\Delta \hat{\omega}_1(t)|^2] = 1.10 \cdot 10^{-7} \), \( E[|\Delta \hat{\omega}_2(t)|^2] = 4.40 \cdot 10^{-7} \) and \( E[|\Delta y(t)|^2] = 7.42 \cdot 10^{-3} \). Note that even though the fundamental frequency \( \omega(t) = \omega_1(t) \) was estimated more accurately, both the joint frequency estimation error \( \Sigma_\omega = E[|\Delta \hat{\omega}_1(t)|^2] + E[|\Delta \hat{\omega}_2(t)|^2] \) and, more importantly, the excess output error were larger than those yielded by (6). This shows clearly that even if the harmonic structure of the interference is known \textit{a priori}, the unconstrained algorithm (6) may be sometimes a better choice than the constrained one.

**REFERENCES**