Design of Robust Steerable Broadband Beamformers with Spiral Arrays and the Farrow Filter Structure

Chiong-Ching Lai, Sven Nordholm, Yee-Hong Leung
Dept. of Computing and Electrical Engineering
Curtin University, Perth, WA, Australia
Emails: c.lai2@postgrad.curtin.edu.au, s.nordholm@curtin.edu.au, y.leung@curtin.edu.au

***Abstract* – This paper extends the design of steerable broadband beamformers by incorporating robustness in the design formulation to cater for microphone mismatches and other non-ideal characteristics. In our method, a 2-norm constraint on the filter weights is imposed in the design optimisation problem. This constraint limits the 2-norm of the filter weights, thus reducing the sensitivity of the beamformer towards the non-ideal characteristics of the microphones. Design examples and comparisons are presented to illustrate our approach.**

Keywords* – *spiral array, Farrow filter, robustness, steerable beamformer

I. INTRODUCTION

Broadband beamformers have become more and more prevalent in commercial applications such as video and audio conferencing, and hands-free communication systems. For some of these applications, tracking of a defined source is vital. The ability to steer the beamformer with a single parameter is thus important and has attracted much research interest. For instance, Parra [1] decouples the spectro-spatial effect of the beampattern in order to steer the main beam by using a Wigner rotation matrix; and, Kajala and Hamalainen [2] use the Farrow filter to design broadband beamformers with single parameter steering capability. The main limitation of [1, 2] is that they do not consider the non-ideal characteristics of the microphones. We will show that these imperfections can lead to severe distortion in the beampatterns.

In order to introduce some degree of robustness into beamformers, mismatches between the design model and practical imperfections must be accounted for. Various studies have proposed different methods to establish robustness against non-deal microphones. Vorobyov et al. [3] optimise the worst case performance by incorporating microphone variations into the design of narrowband beamformers. For broadband beamformers, Doclo and Moonen [4] optimise the mean performance and the worst case criterion after considering the probability density function of the gain and phase of the microphones. However, the authors in [3, 4] do not include the steering ability into their designs. We have employed Doclo and Moonen's white noise gain (WNG) constraint method in [5] into our robust design as it retains the convexity of the original design problem and it also reduces the amplification of uncorrelated noise.

This paper is organised as follows. Section II discusses the concept and idea of steerable broadband beamformers with spiral arrays and the Farrow filter. The robust design extension using norm constraint is then discussed in Section III. In Section IV, design examples are presented and compared to the non-robust beamformer design. The conclusion of this paper is drawn in Section V.

II. MULTI-RING CONCENTRIC SPIRAL ARRAY DESIGN WITH FAR-FIELD SIGNAL SOURCES

A. Array geometry

A multi-ring concentric spiral array with its centre located at the origin of the spatial coordinate system is shown in Fig 1. This is a variant of the one used to measure flow-induced noise in aero-acoustics [6, 7]. It consists of P rings with K microphones uniformly spaced in each ring. Successive rings are offset relative to each other by $2\pi/(KP)$ radians.

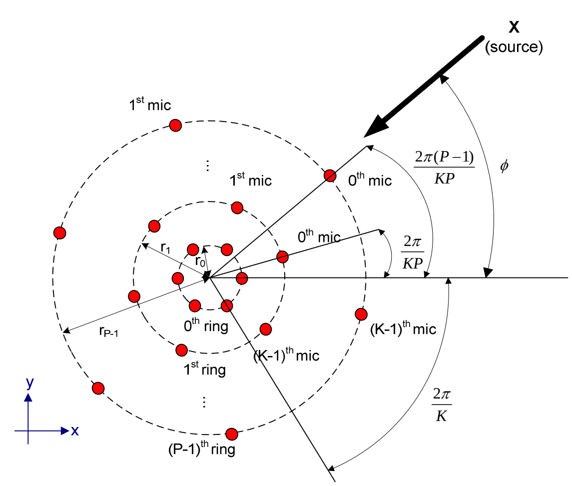

Fig 1 - Microphones placements for multi-ring concentric spiral array

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As can be shown, the delay, $\tau_{p,k}$, of a far-field source signal **x** impinging on the array from azimuth angle ϕ at the k^{th} microphone of the p^{th} ring, relative to the centre of the array is given by (1), for $k = 0,1,...,K-1$ and $p = 0,1,...,P-1$.

$$\tau_{p,k} = \frac{r_p}{c}\cos\left(\phi - \frac{2\pi k}{K} - \frac{2\pi p}{KP}\right) \quad (1)$$

where r_p is the radius of the p^{th} ring and c is the speed of sound in the propagating medium (343*m/s* in air). For a multi-ring array, one will normally let the spacing between adjacent microphones at the innermost ring to be less than half the wavelength of the highest frequency signal in order to avoid spatial aliasing. The inter-microphone spacing at the subsequent rings can be larger to provide sufficient aperture size for the beamformer. Using this idea, we choose the radii of the rings for $p = 0,1,...,P-1$ as follows.

$$r_p = \frac{\alpha c}{2 f_p \sin\left(\frac{\pi}{K}\right)} \quad (2)$$

where $0 < \alpha < 0.5$ to avoid spatial aliasing and $f_p \in \left[f_p^{(1)}, f_p^{(2)}\right]$ are the frequencies of interest in Hertz, selected from within the spectral passband edges described by $f_p^{(1)}$ and $f_p^{(2)}$.

B. Beamformer structure

The desirable characteristics of our beamformer are (i) the ability to steer its main beam easily; and (ii) having a frequency invariant response throughout a wide frequency range, such as the human hearing range. These characteristics can be achieved by designing the beamformer with Farrow filters [2]. The single, real, free parameter in the Farrow filter provides an easy and straightforward means to steer the main beam of the beamformer to a desired look direction. This straightforward steering mechanism enables the beamformer to be steered automatically in real-time when integrated with a source localisation or tracking system. Fig 2 shows the beamformer structure with the Farrow filters.

For generality, the filter weights of the Farrow structure are considered to be complex. The response of the beamformer structure is given by (3).

$$G(\varphi,\omega,\phi) = \sum_{p=0}^{P-1}\sum_{k=0}^{K-1}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} h_{p,k,m}[n] e^{-j\omega n T_s} \varphi^m e^{j\frac{\omega r_p}{c}\cos\left(\phi - \frac{2\pi k}{K} - \frac{2\pi p}{KP}\right)} \quad (3)$$

where T_s is the sampling period, φ is the desired look direction scaled to be within -0.5 to 0.5 inclusively to be used as the steering parameter, $m = 0,1,...,M-1$ is the index of the m^{th} FIR filter of a single Farrow filter and $n = 0,1,...,N-1$ is the discrete time index. (3) can be written in vector form as given by (4).

$$G(\varphi,\omega,\phi) = \mathbf{a}^{\mathrm{H}}(\varphi,\omega,\phi)\mathbf{h} \quad (4)$$

where $\mathbf{a}(\varphi,\omega,\phi) \in \mathbb{C}^{PKMN}$ and $\mathbf{h} \in \mathbb{C}^{PKMN}$ are column vectors with their elements described by (5) and (6) respectively.

$$[\mathbf{a}(\varphi,\omega,\phi)]_v = \varphi^m \exp\left(-j\omega\left(nT_s - \frac{r_p}{c}\cos\left(\phi - \frac{2\pi k}{K} - \frac{2\pi p}{KP}\right)\right)\right) \quad (5)$$

$$[\mathbf{h}]_v = h_{p,k,m}[n] \quad (6)$$

where $v = pP + kK + mM + n + 1$.

C. Non-robust design

In [2], the design of a steerable broadband beamformer is formulated so that its response is matched as close as possible to an ideal brick wall response given by (7).

$$H_d(\varphi,\omega,\phi) = \begin{cases} e^{-j\omega\frac{N}{2}}, & \begin{matrix}\omega_p^{(1)} \le \omega \le \omega_p^{(2)} \\ \varphi - \frac{BW}{2} \le \phi \le \varphi + \frac{BW}{2}\end{matrix} \\ 0, & elsewhere \end{cases} \quad (7)$$

where BW is the beamwidth. The design formulation can be viewed as an optimisation problem that minimises the integral squared error between the actual array response $G(\varphi,\omega,\phi)$ and the ideal response $H_d(\varphi,\omega,\phi)$ as in (8).

minimise

$$E = \int_{\varphi\in\Psi}\int_{\omega\in\Omega_p}\int_{\phi\in\Phi} \left|G(\varphi,\omega,\phi) - H_d(\varphi,\omega,\phi)\right|^2 d\phi\, d\omega\, d\varphi \quad (8)$$

It is sufficient to design the spiral beamformer with its steering region limited within a major sector described by $\Psi \in [-\pi/K, \pi/K]$ in contrast to the entire azimuth angle, such as from $-\pi/2$ to $\pi/2$ in [2]. Steering the look direction outside this major sector can be achieved by simply rotating the sets of designed filter weights to the corresponding microphone. Therefore, rotating the sets of filter weights to the neighbouring microphone is equivalent to incrementing or decrementing the look direction by $2\pi/K$.

The design problem in (8) requires the evaluation of a triple integral. Following Kajala and Hamalainen's method [2], this cost function can be discretised into a finite number of points by discretising the parameters φ, ω and ϕ into S, T and U points respectively, resulting in design problem (9).

minimise

$$\sum_{s=1}^{S}\sum_{t=1}^{T}\sum_{u=1}^{U}\left|\mathbf{a}^{\mathrm{H}}(\varphi_s,\omega_t,\phi_u)\mathbf{h} - H_d(\varphi_s,\omega_t,\phi_u)\right|^2 \quad (9)$$

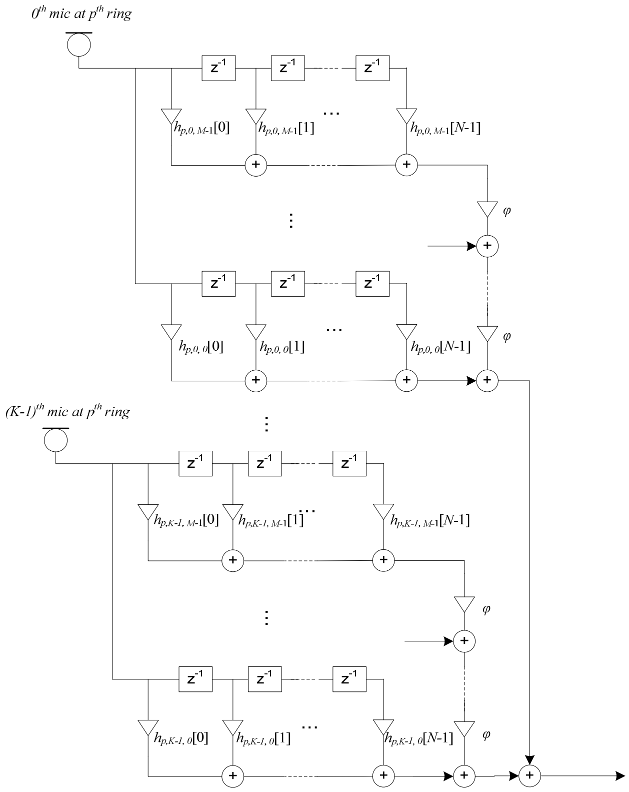

Fig 2 - Steerable multi-ring concentric spiral array with Farrow structure

(9) can be further expressed in matrix form by concatenating horizontally $\mathbf{a}(\varphi_s, \omega_t, \phi_u)$ to form the matrix $\mathbf{A}$ and stacking $H_d(\varphi_s, \omega_t, \phi_u)$ to form the column vector $\mathbf{H}_d$ resulting in (10).

$$\text{minimise} \qquad \left\| \mathbf{A}^{\mathrm{H}}\mathbf{h} - \mathbf{H}_d \right\|^2 \tag{10}$$

(10) can be solved analytically by differentiating the cost function with respect to $\mathbf{h}$ and equating it to zero. (11) gives the analytical solution for the filter weights of the designed beamformer.

$$\frac{d}{d\mathbf{h}}\left(\left\| \mathbf{A}^{\mathrm{H}}\mathbf{h} - \mathbf{H}_d \right\|^2 \right) = 0 \;\; \Rightarrow \;\; \mathbf{h} = \frac{1}{2}\left(\mathbf{A}\mathbf{A}^{\mathrm{H}} \right)^{-1} \mathbf{A}\mathbf{H}_d \tag{11}$$

III. Design of Robust Steerable Broadband Beamformer

The design discussed in Section II.C assumes all microphones to be perfectly aligned to the specified array structure and that they are all ideal and have the same characteristics. The formulation does not consider the mismatches and errors among the microphones. This can cause the designed beamformer to be very sensitive to the non-ideal characteristics of the microphones, which are common in practice.

In order to incorporate robustness against these imperfections into the design formulation, we propose to include a norm constraint on the filter weights in (9). The purpose of the constraint is to prevent the Farrow filter weight from becoming very large. Large filter weights can render the performance of the beamformer to become very sensitive to the non-ideal characteristics of the microphones. The resulting robust design formulation can be further rewritten as a Second Order Cone Programming (SOCP) problem [8]. Define $\mathbf{0}^L$ and $\mathbf{1}^L$ to be $1 \times L$ row vector containing all zeros and ones respectively, and $\mathbf{e} = \left| \mathbf{A}^{\mathrm{H}}\mathbf{h} - \mathbf{H}_d \right|$ to be the absolute error vector. Let $\mathbf{f} = \left[\mathbf{1}^{S \times T \times U} \mid \mathbf{0}^{S \times T \times U} \right]^{\mathrm{T}}$, $\tilde{\mathbf{a}}_{s,t,u} = \left[\mathbf{0}^{S \times T \times U} \mid \mathbf{a}^{\mathrm{T}}(\varphi_s, \omega_t, \phi_u) \right]^{\mathrm{T}}$, $\mathbf{x} = \left[\mathbf{e}^{\mathrm{T}} \mid \mathbf{h}^{\mathrm{T}} \right]^{\mathrm{T}}$ and $\mathbf{c}_{s,t,u}$ be a column vector with unity at w^{th} element, where $w = (s-1)S + (t-1)T + u$, and zeros elsewhere. From (9), the robust beamformer design in SOCP with norm constraint on the filter weights is given by (12), which can be solved using the Yalmip and Sedumi MATLAB packages [9, 10]. Minimising the sum of 2-norm of the errors in (12) is equivalent to minimising the sum of error squared in (9).

$$\begin{aligned} &\text{minimise} && \mathbf{f}^{\mathrm{H}}\mathbf{x} \\ &\text{subject to} && \left\| \tilde{\mathbf{a}}^{\mathrm{H}}_{s,t,u}\mathbf{x} - H_d\left(\varphi_s, \omega_t, \phi_u\right) \right\| \le \mathbf{c}^{\mathrm{H}}_{s,t,u}\mathbf{x} \\ & && \left\| \mathbf{h} \right\| < \beta \\ &\text{for} && s = 1,\dots,S \;,\; t = 1,\dots,T \;,\; u = 1,\dots,U \end{aligned} \tag{12}$$

In (12), the norm constraint limits the norm of the filter weights to be within a specified constant, β . This will reduce the sensitivity of the designed beamformer towards the non-ideal characteristics of the microphones as well as reduce the amplification of uncorrelated noise components. Although there is no clear guideline on the choice of β to trade-off between performance and robustness, Doclo and Moonen [5] did establish a relationship between β and WNG which can be used as a guide.

IV. Design Example

A. *Design of non-robust steerable beamformer with spiral array*

In order to illustrate the non-robust design procedure in Section II.C, a design example with $P = 2$, $K = 5$, $M = 5$, $N = 15$, $\Psi \in [-36°, 36°]$, $\Omega_p \in [500, 3600]\,Hz$ and $\Phi \in [-\pi, \pi]$ are considered. The rings radii are $r_0 = 2.9cm$ and $r_1 = 3.6cm$, which correspond to $f_0 = 3.5kHz$ and $f_1 = 2.8kHz$ in (2) with $\alpha = 0.35$. Fig 3 shows the beampattern for the design with its main beam steered to 20°. It is evident that the main beam maintains a frequency invariant response at the steered direction.

B. *Robust design with norm constraint*

In the case of our robust design procedure, a design example with parameters similar to Section IV.A and $\beta = 5$ will be used. Gain and phase errors following Gaussian distribution with zero means and standard deviations of 0.05

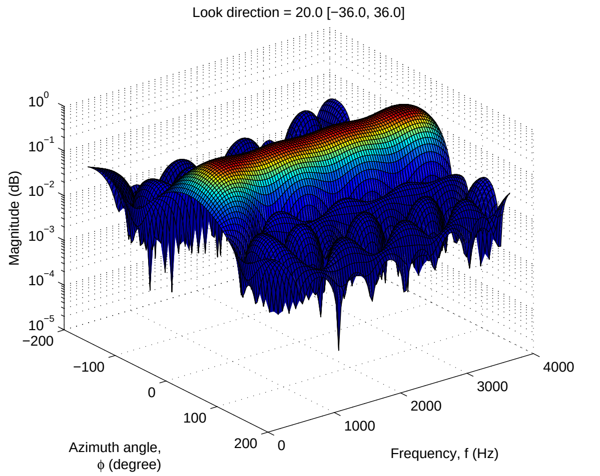

Fig 3 – Beampattern for the steerable broadband beamformer with spiral array, steered to 20°

are introduced (i) into each microphone to illustrate microphone mismatches; and (ii) into each frequency points to illustrate the non-ideal characteristics of the microphones. The beampattern obtained from the non-robust and robust designs are shown in Fig 4 and Fig 5 respectively. Comparison between Fig 3 and Fig 4 shows severe distortion in the beampattern of the non-robust design when gain and phase errors are present. In Fig 5, the robust beamformer still exhibits a clear main beam in the presence of gain and phase errors. Although there is slight distortion at the low frequency part of the beampattern, the distortion is much less than the non-robust design. The WNG for the robust design is 14dB, which is a significant improvement from the non-robust design with WNG of 40dB. From these examples, it is clear that the norm constraint in our design formulation has introduced some degree of robustness against the non-ideal characteristics of the microphones.

V. Conclusion

In this paper, the design of a steerable broadband beamformer using spiral arrays is proposed. The steering capability is made possible by incorporating the Farrow filter structure into the beamformer. The design is also extended to include robustness against the non-ideal characteristics of the microphones by employing a norm constraint on the filter weights. The design example using the robust formulation shows a clear main beam in its beampattern even with gain and phase errors in the microphone elements.

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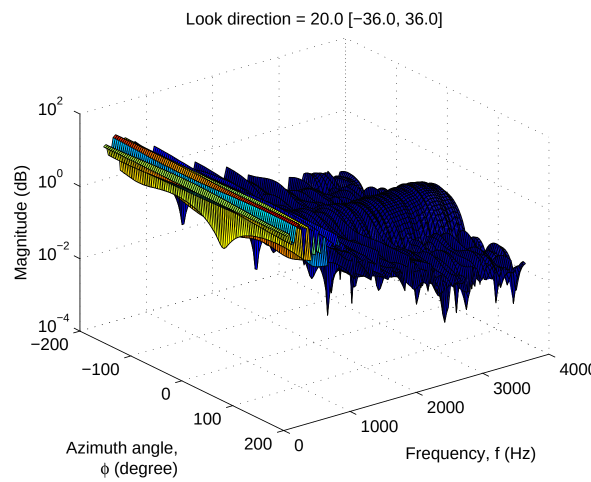

Fig 4 - Beampattern (side view) for the non-robust design steered to 20°, in the presence of gain and phase errors.

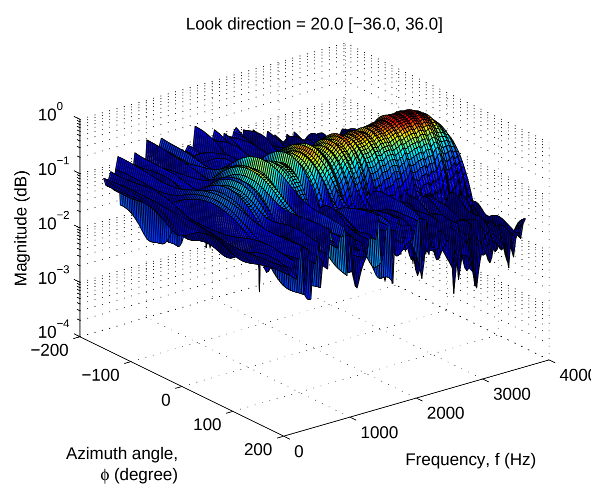

Fig 5 - Beampattern (side view) for the proposed robust design steered to 20°, in the presence of gain and phase errors.