A WEIGHTED OVERLAP-ADD BASED WAVE DOMAIN ADAPTIVE FILTERING ALGORITHM

Paolo Peretti, Laura Romoli, Stefania Cecchi, Lorenzo Palestini and Francesco Piazza

3MediaLabs - Università Politecnica delle Marche
Via Brecce Bianche, 60131 Ancona, Italy
Email: p.peretti@univpm.it

ABSTRACT

The application of adaptive algorithms like noise reduction or echo cancellation on Wave Field Synthesis and Wave Field Analysis may be performed by using Wave Domain Adaptive Filtering in order to limit the computational complexity. The computational cost can be furthermore reduced taking advantage of filter banks theory. Moreover, the signal decomposition into subbands allows to achieve a faster convergence rate and a lower mean-square error for these adaptive algorithms. In this paper we introduce a Weigthed Overlap-Add algorithm for subband processing: it is used to design DFT filter banks where no relation between number of subbands and decimation factor is required. We will show how this scheme leads to the expected results and even to a better behaviour of the transformations involved in Wave Domain Adaptive Filtering.

***Index Terms*—** Wave Domain Adaptive Filtering, Wave Field Synthesis/Analysis, Weighted Overlap-Add.

1. INTRODUCTION

In the last decades, many efforts have been carried out on sound field recording and reproduction techniques based on Huygens principle and on its mathematical representation, given by the Kirchhoff-Helmotz integral. These researches are referred to Wave Field Analysis (WFA) [1] and Wave Field Synthesis (WFS) [2], leading to the definition of WFS and WFA theory: they allow to record the sound field inside a recording room and then to correctly reproduce it in a reproduction room, using an appropariate number of microphones and loudspeakers, positioned along a closed curve (Fig. 1).

A practical implementation (cinemas, theatres, concert halls...) of these concepts to digital signal processing algorithms (room equalization, noise reduction, echo cancellation) needs efficient solutions with the aim of decreasing the high computational cost. To this end, Wave Domain Adaptive Filtering (WDAF) has been introduced [3], as an extension of Frequency Domain Adaptive Filtering (FDAF) [4]. In

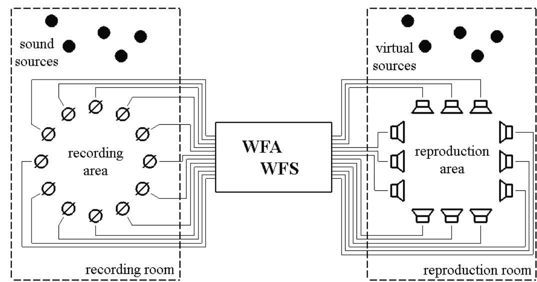

Fig. 1. Typical recording/reproduction setup used with WFA/WFS techniques.

our previuos work, we have presented a subband adaptation scheme of this algorithm, showing its validity in terms of fast convergence and lower mean-square error (MSE) [5].

The signal decomposition into subbands is performed through a filter bank. Many scientific and technological areas are interested on the basic concepts of digital filter banks theory [6], as spectrum analysis, bandwidth compression and radar and sonar processing. This is due to the fact that the subband signal can be better characterized than the full band one. As regards adaptive filtering, the subband decomposition leads to a faster convergence rate and to a lower mean-square error. Moreover, the computational cost of the processing can be reduced. For example, considering an echo cancellation problem, the filters which identify the echo path are generally rather long: the problem may be simplified by dividing the signal into subbands, because more filters of smaller length are identifiable more easily than a long one.

In this paper we propose a subband adaptation scheme based on Weighted Overlap-Add (WOLA) [6]. The paper is organized as follows: first of all, a review of WDAF is presented, then WOLA structure is proposed, showing its effectiveness through some simulation results.

2. WAVE DOMAIN ADAPTIVE FILTERING THEORY

WDAF is the spatio-temporal generalization of FDAF [4, 7]. It is known that FDAF optimal performance arises from the

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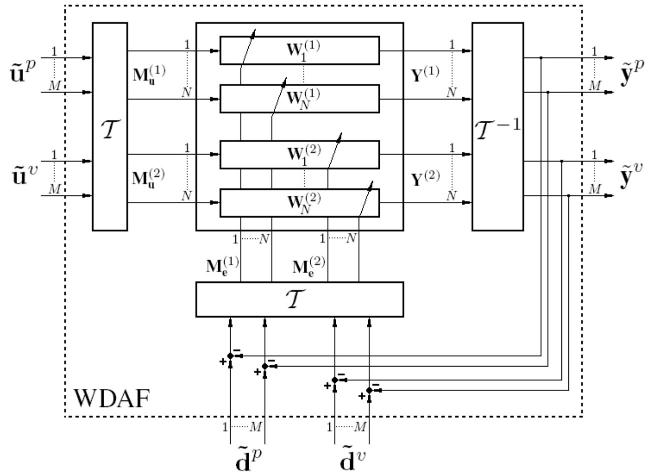

Fig. 2. Typical configuration of WDAF technique.

orthogonality property of the DFT basis functions. Therefore, in order to apply adaptive filtering to WFA/WFS systems, a proper set of orthogonal basis functions has been identified allowing a combined spatio-temporal trasform [3], denoted with $\mathcal{T}$ and $\mathcal{T}^{-1}$.

Assuming to record the sound field with a circular array composed of M pressure and particle velocity microphones [1], the basic scheme of WDAF is shown in Fig. 2, where **u** are the $2M$ input signals, **d** are the $2M$ desired signals, **y** are the $2M$ outputs (in all three cases M pressure and M particle velocity signals are considered) relative to microphone positions and **W** represents the adaptive process composed of $2N$ filters.

The $\mathcal{T}$-transform is obtained by two Fourier transforms $\mathcal{F}_t$, $\mathcal{F}_\theta$ (in temporal and spatial frequency domain) followed by cylindrical harmonics decomposition $\mathcal{M}$ (Fig. 3(a)). In formulae:

$$P(\theta,\omega) = \int_{-\infty}^{\infty} p(\theta,t)e^{-j\omega t}dt \tag{1}$$

$$V_n(\theta,\omega) = \int_{-\infty}^{\infty} v_n(\theta,t)e^{-j\omega t}dt \tag{2}$$

$$\tilde{P}(k_\theta,\omega) = \frac{1}{2\pi}\int_0^{2\pi} P(\theta,\omega)e^{-jk_\theta\theta}d\theta \tag{3}$$

$$\tilde{V}_n(k_\theta,\omega) = \frac{1}{2\pi}\int_0^{2\pi} V_n(\theta,\omega)e^{-jk_\theta\theta}d\theta \tag{4}$$

where (1), (2) represent temporal frequency Fourier transforms, (3), (4) represent spatial frequency Fourier transforms, $p(\theta,t)$ and $v_n(\theta,t)$ are pressure and particle velocity signals recorded by the microphone array, θ is the microphone position angular component in the circular coordinates representation and k_θ is the angular wave number [1]. The expansion coefficients of the sound field in terms of cylindrical harmonics are given by

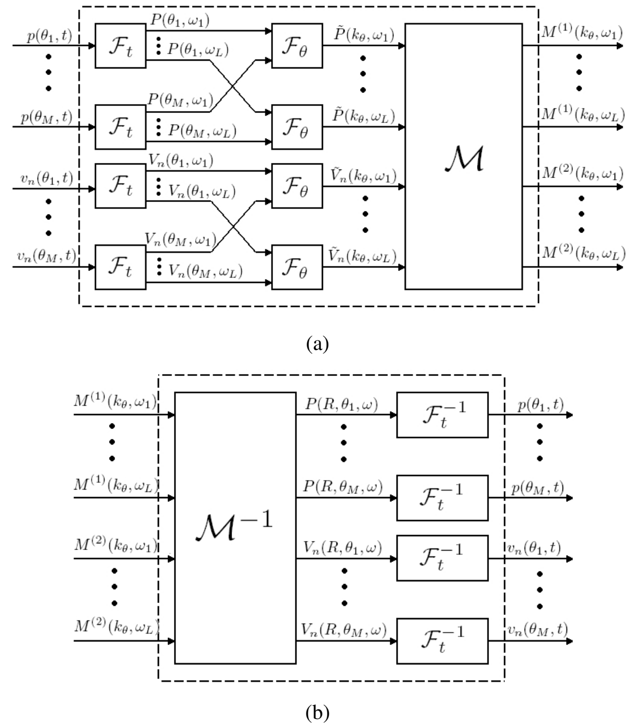

Fig. 3. Wave Domain transform $\mathcal{T}$ (Fig. 3(a)) and inverse transform $\mathcal{T}^{-1}$ (Fig. 3(a)) schemes.

$$M^{(1)}(k_\theta,\omega) = \frac{H_{k_\theta}^{'(2)}(kR)\tilde{P}(k_\theta,\omega) - H_{k_\theta}^{(2)}(kR)j\rho c\tilde{V}_n(k_\theta,\omega)}{H_{k_\theta}^{(1)}(kR)H_{k_\theta}^{'(2)}(kR) - H_{k_\theta}^{(2)}(kR)H_{k_\theta}^{'(1)}(kR)} \tag{5}$$

$$M^{(2)}(k_\theta,\omega) = \frac{H_{k_\theta}^{'(1)}(kR)\tilde{P}(k_\theta,\omega) - H_{k_\theta}^{(1)}(kR)j\rho c\tilde{V}_n(k_\theta,\omega)}{H_{k_\theta}^{(2)}(kR)H_{k_\theta}^{'(1)}(kR) - H_{k_\theta}^{(1)}(kR)H_{k_\theta}^{'(2)}(kR)} \tag{6}$$

where c is the sound velocity, ρ is the medium density, $k = \omega/c$ is the wave number, R is the radius of the microphone array, $H_i^{(j)}$ and $H_i^{'(j)}$ are the Hankel function of kind j and order i and its derivative, respectively.

As for the inverse transform ($\mathcal{T}^{-1}$), we can obtain the temporal frequency domain acoustic field at any point by applying $\mathcal{M}^{-1}$ (Fig. 3(a)):

$$\begin{aligned} P(r,\theta,\omega) = &\sum_{k_\theta} M^{(1)}(k_\theta,\omega)\breve{P}_{k_\theta}^{(1)}(r,\theta,\omega)+ \\ &+\sum_{k_\theta} M^{(2)}(k_\theta,\omega)\breve{P}_{k_\theta}^{(2)}(r,\theta,\omega) \end{aligned} \tag{7}$$

$$\begin{aligned} V_n(r,\theta,\omega) = &\sum_{k_\theta} M^{(1)}(k_\theta,\omega)\breve{V}_{k_\theta}^{(1)}(r,\theta,\omega)+ \\ &+\sum_{k_\theta} M^{(2)}(k_\theta,\omega)\breve{V}_{k_\theta}^{(2)}(r,\theta,\omega) \end{aligned} \tag{8}$$

where

$$\breve{P}^{(1)}_{k_\theta}(r,\theta,\omega) = H^{(1)}_{k_\theta}(kr)e^{jk_\theta\theta} \tag{9}$$

$$\breve{P}^{(2)}_{k_\theta}(r,\theta,\omega) = H^{(2)}_{k_\theta}(kr)e^{jk_\theta\theta} \tag{10}$$

$$\breve{V}^{(1)}_{k_\theta}(r,\theta,\omega) = -\frac{j}{\rho c}H^{'(1)}_{k_\theta}(kr)e^{jk_\theta\theta} \tag{11}$$

$$\breve{V}^{(2)}_{k_\theta}(r,\theta,\omega) = -\frac{j}{\rho c}H^{'(2)}_{k_\theta}(kr)e^{jk_\theta\theta} \tag{12}$$

are cylindrical harmonics referred to pressure and particle velocity field, respectively (r represents the radial component in circular coordinates). Finally an inverse Fourier trasform $\mathcal{F}_t^{-1}$ is needed to come back in the time domain.

A frame-by-frame implementation of these concepts is used [5]. It is based on Fast LMS algorithm [4], where the filters $\mathbf{W}^{(1)}$ and $\mathbf{W}^{(2)}$ are updated in wave domain as follows:

$$\mathbf{W}^{(1)} = \mathbf{W}^{(1)} + \mu\nabla^{(1)} \qquad \mathbf{W}^{(2)} = \mathbf{W}^{(2)} + \mu\nabla^{(2)} \tag{13}$$

where μ is the adaptation stepsize, ∇ is the gradient estimate, normalized to the input power and $^{(1)}$, and $^{(2)}$ are referred to cylindrical harmonics of first and second kind, respectively. In order to obtain a better adaptation of the filters, different μ values could be considered in (13).

3. WOLA STRUCTURE AND ITS APPLICATION TO WDAF

DFT filter banks are efficient systems for the signal decomposition into its spectral components. WOLA is a basic structure for designing DFT filter banks, especially used in the case of not critical downsampling, where no relation between the number of subbands K and the decimation factor P is required [6].

Fig. 4(a) and Fig. 4(b) summarize the analysis and synthesis filter banks, respectively. In accordance with this structure, the filter bank is considered as a block-by-block analysis.

Particular attention has to be given to the project of the proptotype filter h. It is designed in accordance to an iterative least-squares algorithm [8]: given the filter bank reconstruction error ξ_1 and the analysis filters stopband energy ξ_2, the cost function to be minimized is the following:

$$\xi = \xi_1 + \gamma\xi_2 \tag{14}$$

where γ is a positive weighting factor which takes into account the trade-off between the relevance assigned to ξ_1 and ξ_2. The filter length is defined as N_h.

In this paper a blocked-WOLA is used to obtain the subband division. For each microphone, the input signal and the desired signal are bufferized in blocks of length Q. The analysis filter bank is applied to each block, considering P new samples at every iteration. In this way, two matrices $\mathbf{U_m}$ and

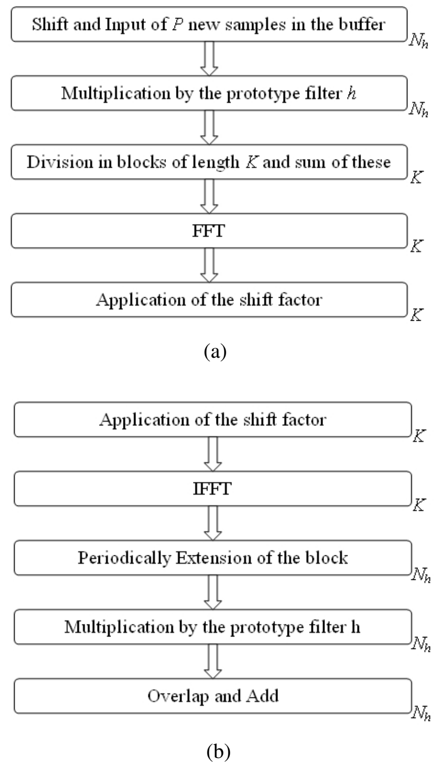

Fig. 4. WOLA structure. Fig. 4(a) and Fig. 4(b) summarize the analysis and the synthesis filter banks, respectively. Pedices represent the length of the buffer at the end of each step.

$\mathbf{D_m}$ are obtained

$$\mathbf{U}_m = \begin{bmatrix} u_{11} & \cdots & u_{1L} \\ \vdots & \ddots & \vdots \\ u_{K1} & \cdots & u_{KL} \end{bmatrix} \mathbf{D}_m = \begin{bmatrix} d_{11} & \cdots & d_{1L} \\ \vdots & \ddots & \vdots \\ d_{K1} & \cdots & d_{KL} \end{bmatrix} \tag{15}$$

where L is the number of bins of each subband and m is the microphone index. Each column corresponds to each output of the analysis filter bank applied to the input buffer with P new samples at every iteration. Each row corresponds to time consecutive bins at the same frequency. Therefore, the subband division is performed considering matrices rows. Overlap and Save with a 50%-overlap is used considering each current row $u_i = [u_{k1}, \cdots, u_{kL}]$ and the previous u_{i-1} to obtain the vector $\tilde{u}_i = [u_{i-1}\ u_i]$. An analogous procedure is applied to $\mathbf{D_m}$. K block-by-block WDAF (Fig. 2) are performed: for each k-th band, a WDAF is applied to the k-th rows relative to all M matrices $\mathbf{U_m}$ and $\mathbf{D_m}$ and the result is stored into the k-th rows of each M matrices $\mathbf{Y_m}$. The output matrices have the same structure as the input ones. In this way, K WDAF shorter than a full band one have to be calculated. For each microphone, the synthesis filter bank is applied to each column of $\mathbf{Y_m}$ to obtain the output signal for each loudspeaker.

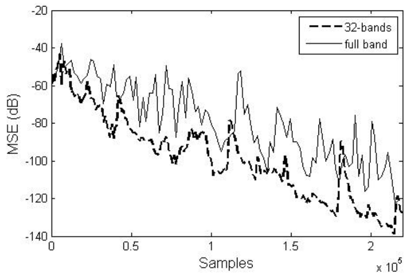

Fig. 5. MSE relative to the pressure signal at the first microphone, for full band and 32 bands cases.

In order to perform consistent comparisons when the number of subbands changes, some parameters have to be chosen in a proper way. Therefore, given the values for K and P, the number of columns L of (15) is multiplied by a factor 2 everytime P is divided by a factor 2. Moreover, in order to process the same number of samples at every iteration, the framesize length Q is derived as $L \times P$. Finally, the prototype filter length has to be adapted to the number of new samples processed by the filter bank analysis. If P increases, a longer filter has to be used. As a consequence, the system introduces a higher delay.

It is worth to underline that the WOLA structure application to the WDAF algorithm has an important adavantage: it is the possibility of choosing a high number of subbands, lowering the computational load as it is well known in literature [6].

3.1. Simulations Results

The proposed and the full band approaches have been applied to an adaptive identification problem. Circular microphones and loudspeakers arrays of radius 1 meter, made up of 16 elements, have been used for the recording and the reproduction of a music source signal positioned outside the arrays. We considered 16 pressure and 16 particle velocity microphones and a sampling frequency of $44,1$ kHz. Fig. 5 shows the MSE relative to the acoustic pressure of the input signal in full band and 32 bands cases for a single microphone: a faster convergence may be observed in the subband case. In order to obtain a better adaptation of the filters, different values for the stepsize μ may be used in each band. Moreover, it is well known in literature that, using the subbands approach, the computational complexity can be considerably reduced with respect to the full band case, but, in order to obtain analysable results, all the transformations involved in the algorithm have to be optimized .

4. CONCLUSION

In this paper we have presented the application of the Weighted Overlap-Add subband structure to Wave Domain Adaptive Filtering. After a brief review on WDAF, the proposed approach has been described and simulations concerning an adaptive identification problem have been presented: tests results of the proposed algorithm have confirmed a reduction of the computational cost. This first result is very interesting given the fact that this kind of algorithm needs a high CPU load in full band condition. In particular Weighted Overlap-Add gives the opportunity of choosing a high number of subbands, decreasing even more the CPU load. Moreover, with a subband approach, a better convergence can be observed in comparison with the full band case. An application of the filter bank in the spatial frequency domain, as well as the one in the temporal frequency domain presented in this paper, could be investigated in future works. Another possible study could be oriented toward the use of extra filters structures in order to cancel the aliasing effects between adjacent bands [9].

5. REFERENCES

[1] E. Hulsebos, D. de Vries, and E. Bourdillat, "Improved microphone array configurations for auralization of sound fields by wave field synthesis," *J. Audio Eng. Society*, vol. 50, no. 10, pp. 779–790, Oct. 2002.

[2] A. J. Berkhout, D. De Vries, and P. Vogel, "Acoustic control by wave field synthesis," *J. Acoust. Soc. Am.*, vol. 93, no. 5, pp. 2764–2778, May 1993.

[3] H. Buchner, S. Spors, and W. Kellermann, "Wave-domain adaptive filtering: Acoustic echo cancellation for full-duplex systems based on wave-field synthesis," *Proc. of IEEE ICASSP*, vol. 4, pp. 117–120, Montreal, Canada, 17-21 May 2004.

[4] J. J. Shynk, "Frequency-domain and multirate adaptive filtering," *IEEE Signal Processing Magazine*, vol. 9, no. 1, pp. 14–37, Jan. 1992.

[5] P. Peretti, L. Palestini, S. Cecchi, and F. Piazza, "A subband approach to wave domain adaptive filtering," *Proc. of Joint Workshop on Hands-free Speech Communication and Microphone Arrays*, Trento, Italy, 05-08 May 2008.

[6] R.E. Crochiere and L.R. Rabiner, *Multirate Digital Signal Processing*, Prentice-Hall, Inc., 1983.

[7] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, Inc., 1996.

[8] M. Harteneck, S. Weiss, and R.W. Stewart, "Design of near perfect reconstruction oversampled filter banks for subband adaptive filters," *IEEE Transactions on Circuits ans Systems*, vol. 46, no. 8, pp. 1081–1085, Aug. 1999.

[9] M.R. Petraglia, R.G. Alves, and P.S.R. Diniz, "New structures for adaptive filtering in subbands with critical sampling," *IEEE Trans. Signal Processing*, vol. 48, no. 12, pp. 3316–3327, Dec. 2000.