

# Multi-channel Active Noise Control for All Uncertain Primary and Secondary Paths

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## Abstract

Fully adaptive feedforward control algorithm is proposed for general multi-channel active noise control (ANC) when all the noise transmission channels are uncertain. To reduce the actual canceling error, two kinds of virtual errors are introduced and are forced into zero by adjusting three adaptive FIR filter matrices in an on-line manner, which can result in the canceling at the objective points. Unlike other conventional approaches, the proposed algorithm does not need exact identification of the secondary paths, and so requires neither any dither signals nor the PE property of the source noises, which is a great advantage of the proposed adaptive approach.

## 1. Introduction

Active noise control (ANC) is efficiently used to suppress unwanted low frequency noises generated from primary sources by emitting artificial secondary sounds to objective points [1][2]. Adaptive feedforward control schemes using the primary noises measured by reference microphones are effective to ANC, since the noise path channels cannot be precisely modeled and are uncertainly changeable. A variety of filtered-x algorithms have been adopted to attain the feedforward adaptation [2][3][4], on the assumption that all the secondary channels are known a priori. To deal with a general case when the secondary path channels are also unknown, almost previous works were based on indirect adaptive approaches which employ on-line identification of the secondary channels. Therefore, dither noise should be added to artificial control sound to assure the PE condition. By taking an extension of the filtered-x type of approaches, the identified models of the secondary channels are used correspondingly in the filtered-x algorithms [3][6][7][8] or the feedforward controller [4][5]. Recently different approaches without secondary path identification have also been proposed [9][10].

The aim of this paper is to propose a fully direct adaptive control approach which does not need explicit identification of the secondary channels. To reduce the actual canceling errors, two kinds of virtual errors are introduced and are forced into zero by adjusting three adaptive FIR

filter matrices in an on-line manner, which enables the noise cancellation at the objective points. Unlike the previous indirect approaches based on the explicit on-line identification, neither dither sounds nor the PE property of the source noises are required in the proposed scheme.

## 2. Feedforward Adaptive Active Noise Control

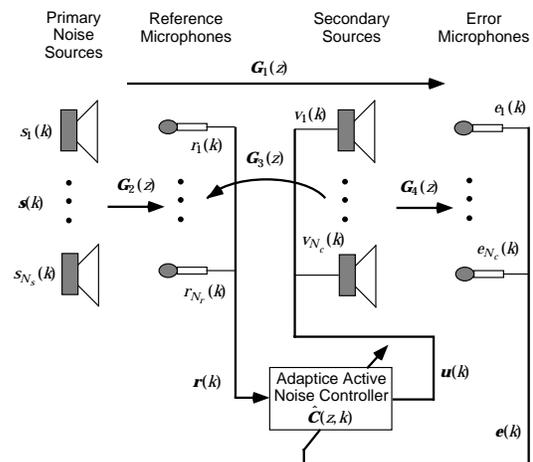


Figure 1: Schematic diagram of multi-channel adaptive feedforward active noise control system.

The setup of multi-channel feedforward active noise control system is illustrated in Fig.1. The primary noises  $s(k) \in R^{N_s}$  are generated from the  $N_s$  sources, and detected by the  $N_r$  reference microphones. The detected signals  $r(k) \in R^{N_r}$  are the input to the  $N_c \times N_r$  adaptive feedforward controller matrix  $\hat{C}(z, k)$ , where  $N_c$  is the number of the secondary loudspeakers which produce artificial sounds to cancel the primary noises at the  $N_c$  objective points. The canceling errors are detected as  $e_c(k) \in R^{N_c}$  by the  $N_c$  error microphones.  $G_1(z) \in \mathcal{Z}^{N_c \times N_s}$  and  $G_2(z) \in \mathcal{Z}^{N_r \times N_s}$  represent the primary channel matrices from the primary noise  $s(k)$  to the reference microphones and error microphones, respectively.  $G_3(z) \in \mathcal{Z}^{N_r \times N_c}$  and  $G_4(z) \in \mathcal{Z}^{N_c \times N_c}$  are the secondary channel matrices from the secondary control sounds  $u(k)$  to the reference and error microphone sets, respectively, where  $G_3(z)$  is referred to as the cou-

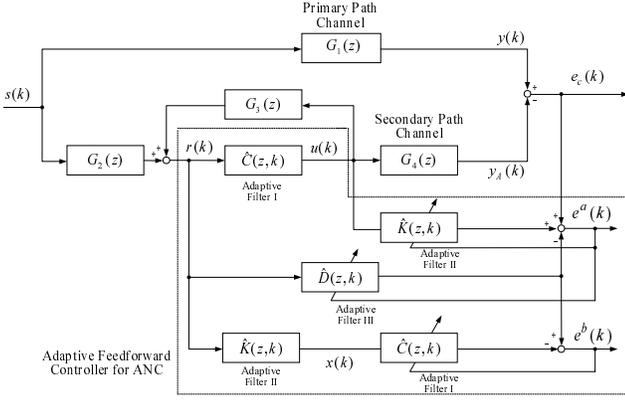


Figure 2: Direct fully adaptive algorithm for single channel ANC

pling channel matrix. All the channels contain model uncertainty and parameter changeability.

It follows from Fig.1 that

$$e_c(k) = \mathbf{G}_1(z)s(k) - \mathbf{G}_4(z)\mathbf{u}(k) \quad (1a)$$

$$\mathbf{r}(k) = \mathbf{G}_2(z)s(k) + \mathbf{G}_3(z)\mathbf{u}(k) \quad (1b)$$

$$\mathbf{u}(k) = \hat{\mathbf{C}}(z, k)\mathbf{r}(k) \quad (1c)$$

where  $\hat{\mathbf{C}}(z, k)$  is an FIR type of adaptive feedforward controller matrix, and the coefficients in  $\hat{\mathbf{C}}(z, k)$  can be updated by various filtered-x adaptive algorithms using the given secondary path models.

### 3. New Adaptive Algorithm for Uncertain Secondary Channels

#### 3.1 Key idea for new adaptation algorithm in single channel case

We give a new direct adaptive algorithm which does not need explicit identification of the secondary path channels, unlike the filtered-x algorithms using the identified model of  $\bar{G}_4(z)$ . The basic structure of the proposed adaptive feedforward control algorithm is illustrated in Fig.2, where  $e_c(k)$ ,  $e^a(k)$ ,  $e^b(k)$  can be expressed as:

$$e_c(k) = \bar{G}_1(z)r(k) - \bar{G}_4(z)u(k) \quad (2a)$$

$$e^a(k) = e_c(k) + \hat{K}(z, k)u(k) - \hat{D}(z, k)r(k) \quad (2b)$$

$$e^b(k) = \hat{D}(z, k)r(k) - \hat{C}(z, k)x(k) \quad (2c)$$

where  $\bar{G}_1(z)$  and  $\bar{G}_4(z)$  are defined in the previous section, and the control input  $u(k)$  and the auxiliary signal  $x(k)$  are also defined as

$$u(k) = \hat{C}(z, k)r(k), \quad x(k) = \hat{K}(z, k)r(k) \quad (3)$$

It follows from Fig.2 and the definitions that

$$\begin{aligned} e^a(k) + e^b(k) &= [e_c(k) + \hat{K}(z, k)u(k) - \hat{D}(z, k)r(k)] \\ &\quad + [\hat{D}(z, k)r(k) - \hat{C}(z, k)x(k)] \\ &= e_c(k) + \hat{K}(z, k)\hat{C}(z, k)r(k) - \hat{C}(z, k)\hat{K}(z, k)r(k) \end{aligned} \quad (4)$$

Thus, if  $e^a(k)$  and  $e^b(k) \rightarrow 0$  for  $k \rightarrow \infty$  is satisfied and the FIR parameters of  $\hat{C}(z, k)$  and  $\hat{K}(z, k)$  converge to any constants, then the second and third terms in the right hand side of (4) can be cancelled, then by the relation  $e^a(k) + e^b(k) = e_c(k)$ , thus it can also be attained that  $e_c(k) \rightarrow 0$  [11].

It seems that  $\hat{D}(z, k)$  and  $\hat{K}(z, k)$  are the identified models for  $\bar{G}_1(z)$  and  $\bar{G}_4(z)$  respectively and the adaptive controller  $\hat{C}(z, k)$  is adjusted according to the identified models of the secondary path dynamics. However, even when the source noise does not satisfy the PE property, the proposed algorithm does not require the convergence of the adjusted parameters to their true values, but only the convergence of their parameters to any constants such that the errors  $e^a(k)$  and  $e^b(k)$  can converge to zero (see Fig.9). Therefore, any dither sounds are not needed unlike the conventional indirect adaptive algorithm. The degradation and complexity caused by the dither signals can also be overcome by the proposed direct adaptive algorithm.

#### 3.2 New ANC algorithm in two-channel case

In a multi-channel case, the exchange of two matrices gives a different result, so the algorithm should be modified. To mitigate the problem, we employ a diagonal matrix as  $\hat{\mathbf{K}}(z, k)$  to derive a fully adaptive algorithm for adjusting the controller parameters. Here for the simplicity of notation, we show the adaptive algorithm in two-channel case.

Similar to the single-channel case, we define two kinds of virtual error vectors  $e^a(k)$  and  $e^b(k)$  which can be described by

$$\begin{aligned} \begin{bmatrix} e_1^a(k) \\ e_2^a(k) \end{bmatrix} &= \begin{bmatrix} e_{c1}(k) \\ e_{c2}(k) \end{bmatrix} + \begin{bmatrix} \hat{K}_{11}(z) & 0 \\ 0 & \hat{K}_{22}(z) \end{bmatrix} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \\ &\quad - \begin{bmatrix} \hat{D}_{11}(z) & \hat{D}_{12}(z) \\ \hat{D}_{21}(z) & \hat{D}_{22}(z) \end{bmatrix} \begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} \end{aligned} \quad (5)$$

$$\begin{aligned} \begin{bmatrix} e_1^b(k) \\ e_2^b(k) \end{bmatrix} &= \begin{bmatrix} \hat{D}_{11}(z) & \hat{D}_{12}(z) \\ \hat{D}_{21}(z) & \hat{D}_{22}(z) \end{bmatrix} \begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} \\ &\quad - \begin{bmatrix} \hat{C}_{11}(z) & \hat{C}_{12}(z) & 0 & 0 \\ 0 & 0 & \hat{C}_{21}(z) & \hat{C}_{22}(z) \end{bmatrix} \begin{bmatrix} x_{11}(k) \\ x_{12}(k) \\ x_{21}(k) \\ x_{22}(k) \end{bmatrix} \end{aligned} \quad (6)$$

where

$$\begin{bmatrix} x_{11}(k) \\ x_{12}(k) \\ x_{21}(k) \\ x_{22}(k) \end{bmatrix} = \begin{bmatrix} \hat{K}_{11}(z) & 0 \\ 0 & \hat{K}_{11}(z) \\ \hat{K}_{22}(z) & 0 \\ 0 & \hat{K}_{22}(z) \end{bmatrix} \begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} \quad (7)$$

$$\begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} = \begin{bmatrix} \hat{C}_{11}(z) & \hat{C}_{12}(z) \\ \hat{C}_{21}(z) & \hat{C}_{22}(z) \end{bmatrix} \begin{bmatrix} r_1(k) \\ r_2(k) \end{bmatrix} \quad (8)$$

In the proposed adaptive algorithm, the coefficient parameters of  $\hat{K}_{ii}(z)$  and  $\hat{D}_{ij}(z)$  are updated so that the

first virtual errors  $e_1^a(k)$  and  $e_2^a(k)$  can be eliminated, while the parameters of  $\hat{C}_{ij}(z)$  are updated so that the second virtual errors  $e_1^b(k)$  and  $e_2^b(k)$  can be eliminated. If these parameters converge to constant and the two virtual errors can be reduced almost to zero, then it follows from (5) ~ (8) that

$$\begin{aligned} e_i^a(k) + e_i^b(k) &= e_{ci}(k) + \hat{K}_{ii}(z)\hat{C}_{i1}(z)r_1(k) \\ &\quad + \hat{K}_{ii}(z)\hat{C}_{i2}(z)r_2(k) - \hat{C}_{i1}(z)\hat{K}_{ii}(z)r_1(k) \\ &\quad - \hat{C}_{i2}(z)\hat{K}_{ii}(z)r_2(k) \quad \text{for } i = 1 \text{ and } 2 \end{aligned} \quad (9)$$

Thus, if the virtual errors  $e_i^a(k)$  and  $e_i^b(k)$  can be reduced to zero and all the adjustable parameters converge to constant, then other four terms in the right hand side can be cancelled and finally it follows that the actual errors  $e_{ci}(k)$  can also be eliminated.

Let the adjustable parameters be the coefficients of the FIR filters defined as:

$$\begin{aligned} \hat{C}_{ij}(z, k) &= \hat{c}_{ij}^{(1)}(k)z^{-1} + \dots + \hat{c}_{ij}^{(m_{ij}^c)}(k)z^{-m_{ij}^c} \\ \hat{K}_{ii}(z, k) &= \hat{k}_{ii}^{(1)}(k)z^{-1} + \dots + \hat{k}_{ii}^{(m_{ii}^k)}(k)z^{-m_{ii}^k} \\ \hat{D}_{ij}(z, k) &= \hat{d}_{ij}^{(1)}(k)z^{-1} + \dots + \hat{d}_{ij}^{(m_{ij}^d)}(k)z^{-m_{ij}^d} \end{aligned}$$

where  $i, j = 1$  and  $2$ . The adaptive parameters  $\{\hat{k}_{ii}^{(m)}(k)\}$  and  $\{\hat{d}_{ij}^{(m)}(k)\}$  are updated so that the instantaneous error norm  $\|e^a(k)\|^2$  may be minimized, while the parameters  $\{\hat{c}_{ij}^{(m)}(k)\}$  are updated so that the error norm  $\|e^b(k)\|^2$  may be minimized.

Let the parameter vectors be defined as

$$\begin{aligned} \hat{\theta}_{Kii}(k) &= (\hat{k}_{ii}^{(1)}(k), \dots, \hat{k}_{ii}^{(m_{ii}^k)}(k))^T \\ \hat{\theta}_{Dij}(k) &= (\hat{d}_{ij}^{(1)}(k), \dots, \hat{d}_{ij}^{(m_{ij}^d)}(k))^T \\ \hat{\theta}_{Cij}(k) &= (\hat{c}_{ij}^{(1)}(k), \dots, \hat{c}_{ij}^{(m_{ij}^c)}(k))^T \end{aligned}$$

and let the regressor vectors be denoted as

$$\begin{aligned} \zeta_i(k) &= [u_i(k-1), \dots, u_i(k-m_{ii}^k)]^T \\ \xi_j(k) &= [r_j(k-1), \dots, r_j(k-m_{ij}^d)]^T \\ \varphi_{ij}(k) &= [x_{ij}(k-1), \dots, x_{ij}(k-m_{ij}^c)]^T \end{aligned}$$

where  $i = 1$  and  $2$ , and  $x_{ij}(k) = \hat{K}_{ii}(z)r_j(k)$ .

Thus the adaptive algorithm for updating these parameters is summarized as follows:

$$\hat{\theta}_{Kii}(k+1) = \hat{\theta}_{Kii}(k) - \gamma_K \zeta_i(k) \varepsilon_i^a(k) \quad (10)$$

$$\hat{\theta}_{Dij}(k+1) = \hat{\theta}_{Dij}(k) + \gamma_D \xi_j(k) \varepsilon_i^a(k) \quad (11)$$

$$\begin{aligned} \varepsilon_i^a(k) &= e_i^a(k) / [1 + \gamma_D (\|\xi_1(k)\|^2 + \|\xi_2(k)\|^2) \\ &\quad + \gamma_K \|\zeta_i(k)\|^2] \end{aligned} \quad (12)$$

$$\hat{\theta}_{Cij}(k+1) = \hat{\theta}_{Cij}(k) + \gamma_C \varphi_{ij}(k) \varepsilon_i^b(k) \quad (13)$$

$$\varepsilon_i^b(k) = \frac{e_i^b(k)}{1 + \gamma_C (\|\varphi_{i1}(k)\|^2 + \|\varphi_{i2}(k)\|^2)} \quad (14)$$

where  $i = 1$  and  $2$ .

Other adaptive algorithms can also be derived, for instance, the norm  $\|e(k)\|^2$  of the augmented error  $e(k) = (e^{aT}(k), e^{bT}(k))^T$  can also be minimized, which gives an alternative algorithm which requires more computation. Therefore, we adopted the proposed algorithm given by (10) to (14) in the experiment.

## 4. Simulation Results

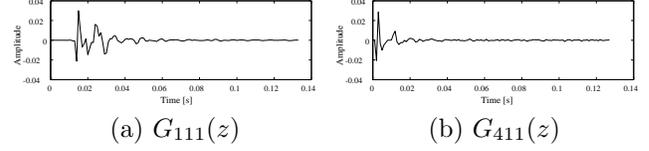


Figure 3: Example of identified FIR channels

Fig.3 depicts examples of FIR models of the primary and secondary channels which were identified by experiments for the active noise control in a room. These models are used in the simulations. Two primary loudspeakers, two reference microphones, two secondary loudspeakers, and two error microphones are placed initially at specific locations, which configures a two-channel active noise control system ( $N_s = N_r = N_c = N_e = 2$ ). The sampling period is chosen 1 ms. The power spectra of the primary source noises  $s_1(k)$  and  $s_2(k)$  are limited in low frequency range from 50 to 400 Hz, or have sometimes sinusoids with unknown frequencies which do not satisfy the PE condition. Therefore in the conventional indirect approaches need dither sounds which are additively generated from the secondary loudspeakers to identify the secondary channels to assure the PE condition. On the other hand, the proposed approach does not need any dither sounds and the PE condition, but it can attain the noise attenuation even in the presence of uncertainties of the secondary channels.

Figs.4 to 6 show the actual canceling errors  $e_{c1}(k)$  and  $e_{c2}(k)$ . In the numerical simulations, the locations of the two error microphones are changed by 34[cm] instantaneously far from the original positions by using the switches at 10[s] after the start of control. This causes the uncertain changes of the secondary path channels, and the extended error based filtered-x algorithm [5] could not keep the stable attenuation performance as shown in Fig.5, since it needs precise knowledge on the secondary channels. On the other hand, the proposed method can attain the stable control performance even if the secondary channels change very rapidly. Thus, the proposed algorithm does not need on-line identification of the secondary channels.

Figs.7 and 8 show the control results and Fig.9 shows one of parameters of  $\hat{D}_{11}(z, k)$  and  $\hat{K}_{11}(z, k)$ , when the primary source noises consist of sinusoids with unknown

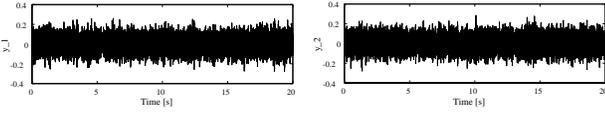


Figure 4:  $e_{c1}(k)$  and  $e_{c2}(k)$  in case without control.

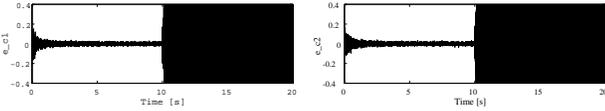


Figure 5:  $e_{c1}(k)$  and  $e_{c2}(k)$  obtained by the extended error based filtered-x algorithm (Shimizu *et al.*, 2002).

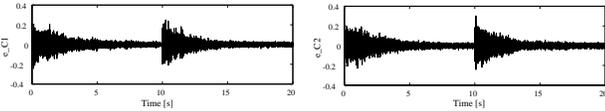


Figure 6:  $e_{c1}(k)$  and  $e_{c2}(k)$  obtained by the direct fully adaptive algorithm.

frequencies (actually about 150 Hz and 250 Hz in the interval (0s, 4s), and 100 Hz in the interval (8s, 10s)). In Fig.9, dotted lines indicate 16th coefficient of  $G_{111}(z)$  (left figure) and 4th coefficient of  $G_{411}(z)$  (right figure). Even when the primary source noises have an insufficient PE condition of the primary source noises like sinusoids, the proposed algorithm can give very nice attenuation performance in the interval (0s, 4s), and  $d_{11}^{16}(k)$  and  $k_{11}^4(k)$  converged to any constants. During the interval, the adaptive algorithm updates only small number of parameters for reducing the canceling errors. During the interval (4s, 8s), the primary source noises have rather much PE property and then the many parameters of the adaptive filters should be updated, so the convergence time is need to achieve nice attenuation performance. During the interval (8s, 10s) the primary source noises are sinusoids again, however, since the adjustment of almost all adaptive parameters has been completed, then no parameters are required to update. Thus, the proposed control scheme is very robust to the insufficiency of the primary source noises unlike the conventional approaches.

## 5. Conclusion

The proposed fully direct adaptive control approach can work effectively even when all of the primary and secondary channel dynamics are uncertainly changeable. To reduce the canceling error, two virtual errors are introduced and are forced into zero by adjusting the three adaptive FIR filter matrices in an on-line manner, which enables the noise cancellation at the objective points. Unlike the previous methods, neither dither noises nor the PE property of the source noises are required.

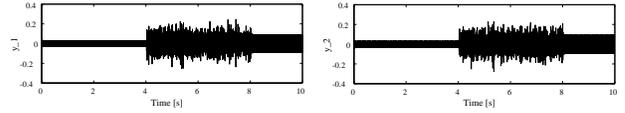


Figure 7:  $e_{c1}(k)$  and  $e_{c1}(k)$  in case without control

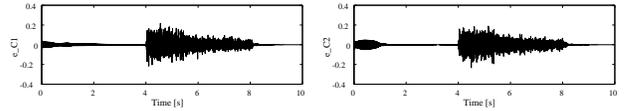


Figure 8:  $e_{c1}(k)$  and  $e_{c2}(k)$  obtained by the proposed fully adaptive algorithm

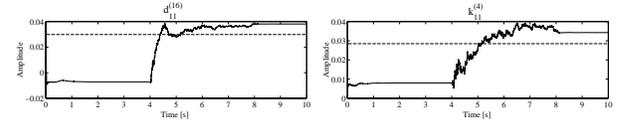


Figure 9: Parameters  $d_{11}^{(16)}(k)$  and  $k_{11}^{(4)}(k)$  in  $\hat{D}_{11}(z, k)$  and  $\hat{K}_{11}(z, k)$ , respectively.

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