ADAPTIVE BOOLEAN FILTERS FOR THE PREDICTION STAGE IN LOSSLESS IMAGE COMPRESSION

Doina Petrescu and Moncef Gabbouj

Signal Processing Laboratory, Tampere University of Technology
P.O. Box 553, FIN-33101 Tampere, Finland
doina@cs.tut.fi

ABSTRACT

This paper proposes adaptive Boolean filters for realizing the prediction stage in lossless compression of greylevel images. The adaptation of Boolean filters is performed using local information about the presence and orientation of edges (high local amplitude variations) of the pixels inside the prediction mask. It is shown that adaptive predictors improve the performance of optimal Boolean predictors, and they outperform other recently proposed nonlinear predictors: the median adaptive predictor [2] and the gradient adjusted predictor [9]. Finally, the gradient adaptive Boolean predictor is included in a lossless compression scheme, and the compression results are presented.

1. INTRODUCTION

Boolean filters [1] have been efficiently used in image restoration, mainly for images corrupted by noise with impulsive characteristics. Fast procedures [1], [7] were proposed to design the optimal filter for a given set consisting of the target and input images. Recently, optimal Boolean filters have been successfully used in a novel application, namely achieving the prediction task for lossless image compression [5]. Simple and block-adaptive prediction structures using optimal Boolean and FIR–Boolean hybrid filters have been investigated and shown to outperform different linear and nonlinear prediction schemes.

Prediction is generally the first step in lossless compression, aiming to decorrelate the image prior to encoding. Common predictors produce low errors in smooth areas in images, and large errors near edges. One method to reduce the prediction errors near edges is to use an adaptive predictor, which changes in the presence of edges, according to the orientation and amplitude of the gradient. A predictor depending on the local gradient, namely the 'gradient adjusted predictor', was used by the recently proposed CALIC system [9]. A recent comparative study [3] showed that it achieves the best prediction performance in terms of the lowest entropy for prediction errors, among the different predictors included in the proposals for a new lossless image compression standard.

In this paper we extend the study of the performance of Boolean predictors for lossless image compression, proposing an adaptive prediction scheme, where the Boolean predictor changes based on the local gradient amplitude between different predictors for the contexts defined by the shape of the greylevel discontinuity. We found that the optimal Boolean filters used for prediction for each context achieve lower entropy for prediction errors than in the case when one optimal filter is used for prediction for the whole image. However, encoding optimal predictors for each context overloads the side information and reduces the compression ratio. Aiming to keep the side information small, we develop an encoding/decoding scheme, where the predictors for different local contexts are adaptively designed while processing the data.

2. BOOLEAN GRADIENT ADAPTIVE PREDICTION

Consider an $n_r \times n_c$ greylevel image, with pixel values in $\{0,\ldots,M\}$ the current pixel being denoted $D(i,j)$. The values of N pixels in a causal neighborhood of the current pixel, $\mathcal{N}_{D(i,j)}$, enclosed in the input vector $\underline{X}(i,j)$ are used to calculate the predicted value $\hat{D}(i,j)$, as the output of a Boolean filter, as follows:

$$\hat{D}(i,j) = \sum_{m=1}^{M} f(T_m(\underline{X}(i,j))), \tag{1}$$

where $(i,j) \in \{1\ldots n_r\} \times \{1\ldots n_c\}$, f denotes the Boolean function of the predictor, and T_m is the thresh-

On leave from Polytechnic University of Bucharest, Department of Electronics and Telecommunication, Splaiul Independentei 313, R-77206 Bucharest, Romania.

olding operator

$$T_m(D) = \begin{cases} 1, & \text{if } D \geq m \\ 0, & \text{else,} \end{cases} \qquad (2)$$

applied to each component of the input vector $\underline{X}(i,j)$. An equivalent computation of the output is performed using only N threshold values for the input vector, namely the values of its components, as follows:

$$\begin{aligned} \hat{D}(i,j) &= X_{(1)} f([1 \ \dots \ 1]) \\ &+ \sum_{m=1}^{N-1} (X_{(m+1)} - X_{(m)}) f(T_{X_{(m+1)}}(\underline{X}(i,j))) \\ &+ (M - X_{(N)}) f([0 \ \dots \ 0]), \end{aligned} \qquad (3)$$

where $X_{(1)} \leq \ldots \leq X_{(N)}$ denote the ordered elements of the input vector. We denote the prediction errors by

$$e(i,j) = D(i,j) - \hat{D}(i,j). \qquad (4)$$

The optimal Boolean predictor is designed in a first pass through the image to be encoded, using the procedure proposed in [7]: a set of 2^N statistics are computed using comparisons and accumulations, and then the Boolean function values are decided based on the sign of these statistics. The Boolean function, in the form of a 2^N length vector is the side information for encoding the optimal predictor.

In the gradient adaptive prediction method, different optimal Boolean predictors are used for different contexts distinguished by the local greylevel discontinuity. At each pixel the presence of a greylevel discontinuity inside the prediction mask is detected using the difference between the local maximum and the local minimum:

$$\Delta(i,j) = \max(\underline{X}(i,j)) - \min(\underline{X}(i,j)). \qquad (5)$$

The local gradient is compared to a threshold T_Δ, which can be designed for each image based on the distribution of the local gradient in the first pass and was empirically set to 8 in our experiments on 8-bits images. If the gradient value is lower than the threshold one predictor is used, otherwise the predictor is switched between several predictors based on the local information about the shape or orientation of the discontinuity. Two types of contexts are used in our experiments for describing the shape of the discontinuity.

(a) The local maximum and local minimum positions inside the processing mask.

(b) The binary pattern obtained by thresholding the values inside the prediction mask at the average between the local maximum and the local minimum.

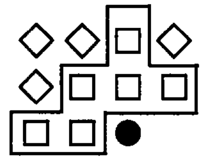

Figure 2: 6-pixels (squares) discontinuity detection mask for switching between 10-pixel Boolean predictors (squares and diamonds) to predict the black pixel.

For each context, an optimal Boolean filter designed using the data of that context is used for prediction. In order to keep the side information low, in our prediction procedure only the overall Boolean optimal predictor, computed in the first pass through the image is encoded and it is used for smooth regions, where the gradient value is lower than the threshold. The Boolean predictors for the different contexts when the gradient value is high are adaptively designed along the data processing flow. After encoding each pixel for which a discontinuity was detected, the Boolean predictor for the current pixel context is updated. This is achieved by updating the statistics corresponding to the current prediction mask. At each pixel, at most $N+1$ statistics are updated, and the corresponding Boolean function values are decided. The encoding procedure for a total number of K different contexts for adaptive prediction is briefly summarized in Figure 1.

3. EXPERIMENTAL RESULTS

To test the performance of the proposed adaptive Boolean prediction procedure we have encoded a set of eight 576×720 images from the JPEG test set. We have used for prediction a 10-pixel mask, and for detecting local discontinuities only a 6-pixel mask included in the prediction mask. This option was made in order to reduce the number of different contexts. There are 30 different contexts when the positions of local maximum and local minimum are used for switching and at most 63 contexts when the binary pattern is used for switching. In Figure 2, the pixels of the discontinuity detection mask are denoted by squares, and the prediction mask contains in addition to the discontinuity detection mask the 4 pixels denoted by diamonds.

The performance of the proposed adaptive prediction scheme compared to the median adaptive predictor proposed in [2], the gradient adjusted predictor [3], and optimal Boolean and FIR Boolean hybrid predictors [5] for a set of test images is presented in Table 1.

Step 1.

Encoder: Design the optimal Boolean predictor f^0 for the whole image in a first pass through the image, and encode f^0.

Decoder: Decode the Boolean predictor f^0.

Step 2. Initialize the K sets of 2^N statistics for the adaptive Boolean predictors and set the corresponding Boolean functions f^k, $k \in \{1, \ldots K\}$ values to 'undecided'.

Step 3. At each pixel $(i,j) \in \{1 \ldots n_r\} \times \{1 \ldots n_c\}$

- Calculate the local gradient $\Delta(i,j)$ using (5), to decide upon the predictor.
- If $\Delta(i,j) > T_\Delta$ find the local context for the discontinuity $k \in \{1, \ldots K\}$ and set the Boolean predictor f to f^k, else set the Boolean predictor f to f^0.
- Sort the values inside the processing mask and calculate the set of binary vectors obtained by thresholding the input vector $\{T_m(\underline{X}(i,j)),\ m \in \{X_{(1)}, \ldots X_{(N)}, M\}\}$.
- Calculate the predicted value $\hat{D}(i,j)$ using (3). Whenever the 'undecided' value for the Boolean function is encountered for some binary vector, use the value of f^0 instead of f^k for that vector.
- **Encoder**: Calculate the prediction error $e(i,j)$ using (4) and encode this error.
  Decoder: Decode the error $e(i,j)$ and calculate the real value $D(i,j) = \hat{D}(i,j) + e(i,j)$.
- If $\Delta(i,j) > T_\Delta$, update the statistics of the current context k for the set of binary vectors obtained from thresholding the input vector, and decide the Boolean function f^k for these vectors as in [7].

Figure 1: Encoding/decoding procedure using gradient adaptive Boolean filters for prediction

Image	JPEG	Boolean(10)	EBA$_p$	EBA$_b$	MedAP	GAP
Balloon	3.172	3.100	3.055	3.010	3.122	3.046
Barb1	5.302	5.007	4.967	4.885	5.208	5.137
Barb2	5.236	5.056	5.015	4.990	5.162	5.190
Boats	4.469	4.293	4.222	4.198	4.313	4.286
Girl	4.225	4.108	4.094	4.042	4.318	4.103
Gold	4.875	4.708	4.679	4.666	4.717	4.676
Hotel	4.943	4.637	4.578	4.596	4.735	4.661
Zelda	4.179	3.964	3.932	3.903	4.106	3.951
Average	**4.550**	**4.359**	**4.318**	**4.286**	**4.450**	**4.363**

Table 1: Entropy of errors for different predictors: JPEG best linear predictor, optimal Boolean predictor for 10-pixel prediction mask, gradient adaptive Boolean predictors with contexts from the positions of local maximum and local minimum (EBA$_p$) and from binary pattern (EBA$_b$), median adaptive predictor (MedAP) and gradient adjusted predictor (GAP).

4. LOSSLESS IMAGE COMPRESSION SCHEME

A compression scheme, incorporating Boolean adaptive predictor was used for the lossless compression of images. It was shown [3] that the prediction stage itself cannot completely remove all the correlations between adjacent pixels. The compression performance can be improved by using conditional source models with finite memory for encoding the prediction errors [8]. Different types of contexts have been used in image coding for conditioning the probabilities of the source model, in order to minimize the optimal code [9], [6]. In our scheme we used for context modeling the amplitude of the gradient inside the prediction mask. By conditioning the error distribution on Δ, the prediction errors are clustered into classes with different probability distributions. In smooth areas, for small values of the local gradient, small prediction errors have the highest probabilities, while in areas where large greylevel variations are present larger prediction errors are more frequent. We quantize the values of this gradient to $L = 12$ levels, and 12 different contexts are used to condition the error probabilities. The quantization levels may be designed in the first pass through the image, based on the histogram of the local gradient. The values of these levels are then encoded as side information. In our experiments, we noticed that a good choice for the quantization levels is:

$$T = \{0, 3, 6, 9, 12, 18, 28, 40, 55, 70, 90, 120, 256\}$$

The contexts are defined as intervals for the values of the gradient, such that for C_ℓ , $T(\ell) \leq \Delta < T(\ell+1)$, $\ell \in \{1, \ldots L\}$. A figure of the compression achieved by this modeling is the conditional entropy of prediction errors computed as in [6]:

$$-\sum_{\ell=1}^{L} \sum_{e=-M}^{M} p(e, C_\ell) \log_2 p(e|C_\ell) \tag{6}$$

where the probabilities are evaluated based on the cardinalities of the following sets of pixels

$$\begin{aligned} S_{e,\ell} &= \{(i,j)|e(i,j) = e, T_\ell \leq \Delta(i,j) < T(\ell+1)\} \\ Q_\ell &= \bigcup_{e=-M}^{M} S_{e,\ell}. \end{aligned} \tag{7}$$

Then

$$\begin{aligned} p(e, C_\ell) &= \frac{\mathrm{Card}(S_{e,\ell})}{n_r \times n_c} \\ p(e|C_\ell) &= \frac{\mathrm{Card}(S_{e,\ell})}{\mathrm{Card}(Q_\ell)}. \end{aligned} \tag{8}$$

Image	Entropy of errors	Bit-rate
Balloon	2.885	2.968
Barb1	4.503	4.596
Barb2	4.620	4.716
Boats	3.915	4.001
Girl	3.842	3.931
Gold	4.476	4.557
Hotel	4.333	4.430
Zelda	3.780	3.869
Average	**4.044**	**4.133**

Table 2: Coditional entropy of errors after contextual error modeling and bit-rate after arithmetic encoding

The prediction errors are encoded by an m-ary arithmetic coder using the 12 contexts. The arithmetic coder is a public domain package based on the work developed in [4].

The conditional entropy of prediction errors and the bitrate after arithmetic encoding for the images in the test set are presented in Table 2, when the gradient adaptive Boolean predictor is used for prediction. An important reduction in the bitrate is achieved by contextual error modeling of the prediction errors.

5. CONCLUSIONS

Adaptive Boolean predictors based on the amplitude and orientation of local greylevel discontinuities have been introduced. A procedure for designing the adaptive predictors along the data processing flow was proposed. The entropy of prediction errors is reduced without increasing the side information for predictor encoding. The adaptive predictor was incorporated in a lossless compression scheme for coding greylevel images, where gradient based context modeling of prediction errors was performed to further reduce the entropy.

6. REFERENCES

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