Nonlinear Filter Design Based on Fuzzy Inference Rules for Image Processing

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Abstract

A fuzzy filtering system is proposed for image restoration. Since all the conventional filters have their specific characteristics, they act well for some environments but with poor performance for others. The fuzzy filtering system gives the method to aggregate the advantage of the conventional filters to obtain the improved performance for image restoration.

1 Introduction

In this paper, a fuzzy filtering system which gives the method to aggregate the advantage of the conventional filters to obtain the improved performance for image restoration is proposed.

Each of the famous class of filters has its special characteristics. They may do very well for some specific environment but not all the environments. For example, the median filters are useful for constant region but bad for edges. The *RCRS* filters can not do well in descending or ascending regions while the α-trimmed mean filters do very well on them [1]. Moreover, when the noise rate is low, most of the signals are uncorrupted. Thus, the detection scheme is the good choice. It motivates us to aggregate the advantage of the important conventional filters by fuzzy inference rules so that the filtering work can be done by appropriate filters to obtain high filtering performance in various environments. Since the system is based on the fuzzy inference rules, we call it the *fuzzy filtering system.*

There are some researches related to this problem. The fuzzy rules are used to make the decision for passing the samples or filtering it with median filters [2]. And, in [3], a noise detection algorithm is proposed and the output value is determined by the choice of the center sample or the output of some classical filter which is one of the conventional filters. It takes the advantage when noise ratio is low. Whereas, when the noise ratio is high, for example 30% noise, the detection scheme will be poor. In our research, the advantage of the conventional filters are aggregated by fuzzy inference rules. There are many rule-of-thumbs for filtering work. These rule-of-thumbs or experiences can be used in knowledge base of the fuzzy filtering system. Thus, the appropriate filter will be used during the filtering process to get better performance.

2 Fuzzy Filtering System

The general architecture of the fuzzy filtering system is shown in Figure 1. It is obtained from the modification of the *Takagi and*

Sugeno's fuzzy controller [4]. The *fuzzifica-*

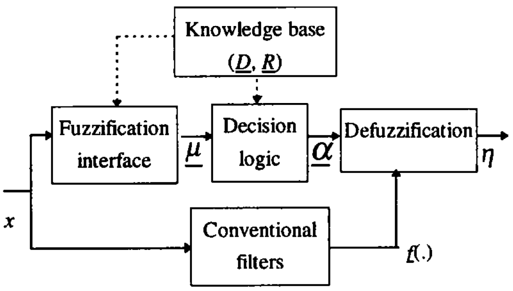

Figure 1: The architecture of fuzzy filtering system.

tion interface includes two functional modules which are shown in Figure 2. The *attribute ab-*

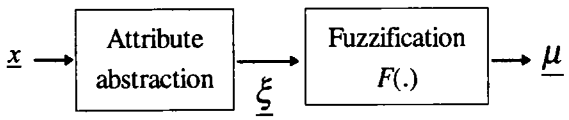

Figure 2: The functional modules of the fuzzification interface.

straction module receives the current observation vector $\underline{x}$ and abstracts the *attributes* (or *features*) of the observation vector, which is denoted as $\underline{\xi}=(\xi_1, \xi_2, ..., \xi_m)$. The attributes are mapped into suitable input domains of the *fuzzification* module $F(.)$, that is, the attribute ξ_i, i =1, 2, ..., m, is a value in one of the input domains of $F(.)$. Each of the input domains of $F(.)$ can be partitioned into some fuzzy sets. For example, the *variation* which is defined to be the different between maximum and minimum of the elements of the observation vector can be viewed as an input domain of $F(.)$. It is partitioned into three fuzzy sets, i.e., $\mu_1^{(1)}$(=*low-variation*), $\mu_1^{(2)}$(=*middle-variation*) and $\mu_1^{(3)}$(=*high-variation*). Thus, the observation vector $\underline{x}$ can be converted into a linguistic term (for example, *low-variation)* or a fuzzy set (for example, $\mu_1^{(1)}$). Let the input domains be χ_1, χ_2, ..., and χ_m. We define p_1 distinct fuzzy sets $\mu_1^{(1)}, \mu_1^{(2)}, ..., \mu_1^{(p_1)} \in F(\chi_1)$ on the set χ_1. They are associated with p_1 distinct linguistic terms, i.e., $A_1^{(1)}, A_1^{(2)}, ..., A_1^{(p_1)}$. In the same way, each of the domains χ_2, ..., and χ_m are partitioned into p_i fuzzy sets $\mu_i^{(1)}, \mu_i^{(2)}, ..., \mu_i^{(p_i)} \in F(\chi_i)$, i =2, ..., m, respectively. They are associated with p_i distinct linguistic terms, i.e., $A_i^{(1)}, A_i^{(2)}, ..., A_i^{(p_i)}$, i =2, ..., m, respectively. The output of *fuzzification interface* is $\underline{\mu}=(\mu_1(\xi_i), \mu_2(\xi_i), ..., \mu_m(\xi_i))$ where

$$\mu_i(\xi_i) = \begin{bmatrix} \mu_i^{(1)}(\xi_i) \\ \mu_i^{(2)}(\xi_i) \\ \vdots \\ \mu_i^{(p_i)}(\xi_i) \end{bmatrix} \quad (1)$$

$$= (\mu_i^{(1)}(\xi_i), \mu_i^{(2)}(\xi_i), ..., \mu_i^{(p_i)}(\xi_i))^t \quad (2)$$

where ξ_i, i =1, 2, ..., m, is the ith attribute of the observation vector $\underline{x}$. Let's see the following example.

The knowledge base of the fuzzy filtering system is formed by data base $\underline{D}$ and rule base $\underline{R}$. The fuzzy partitions and the linguistic terms associated with fuzzy sets form the data base of the knowledge base. Now, we specify the rule base $\underline{R}$ as follows.

Suppose that each of the *input variables* ξ_i, $i = 1, 2, ..., m$, is a value in input domain χ_i. The rule base consists of k control rules, that is,

$$R_r : \text{if } \xi_1 \text{ is } A_{1,r}^{(v_1)} \text{ and ... and } \xi_m \text{ is } A_{m,r}^{(v_m)} \text{ then } \eta \text{ is } f_{\beta_r}(\underline{x}),\ r = 1, 2, ..., k, \quad (3)$$

where $A_{i,r}^{(v_i)}$, $v_i \in \{1, 2, ..., p_i\}$, i =1, 2, ..., m, is the v_ith fuzzy partition in χ_i which has p_i partitions. Each of the functions f_{β_r}, r=1, 2, ..., k, represents a conventional filters. Thus, when a fuzzy inference rule is activated, the corresponding conventional filter is used to do the filtering work. Note that the indices of the output functions β_1, β_2, ..., β_k may not be distinct, i.e., some rules may be corresponding to the same conventional filter.

The decision logic is the same as the *Takagi and Sugeno's fuzzy controller* in which it determines the degree of applicability of each of the rules R_1, R_2, ..., R_k in the rule base. The degree of applicability of rule R_r is defined to be $\alpha_r = \min\{\mu_1^{(v_1)}(\xi_1),\ \mu_2^{(v_2)}(\xi_2),\ ...,\ \mu_m^{(v_m)}(\xi_m)\}$ where $\mu_i^{(v_i)}$, $i = 1, 2, ..., m$, is the membership function of the v_ith fuzzy partition in the ith input domain χ_i. The operator "min" may be extended to be a T-norm which is still an important research topic [5, 6].

Finally, in the block of defuzzification, the output of this filtering system η can be obtained as follows.

$$\eta = \Phi(\alpha_1 \cdot f_{\beta_1}(\underline{x}),\ \alpha_2 \cdot f_{\beta_2}(\underline{x}),\ \cdots,\ \alpha_k \cdot f_{\beta_k}(\underline{x})) \tag{4}$$

where Φ denotes an averaging operator or an S-norm operator [6], $f_{\beta_r}(\underline{x})$ is a conventional filter which may be linear or nonlinear and α_r is the degree of applicability of each rule R_r. For simplicity, the linear combination

$$\eta = \frac{\sum_{r=1}^{k} \alpha_r \cdot f_{\beta_r}(\underline{x})}{\sum_{r=1}^{k} \alpha_r} \tag{5}$$

is used as the output formula. From Eq. (5), it is easy to see that the output of this filtering system is a combination of some conventional filters. The coefficients α_r, r=1, 2, ..., k, are determined by the knowledge base of filtering system which is obtained from the knowledge (or experiences) of human experts. Thus, the characteristics of conventional filters can be successfully aggregated in this filtering system and the filtering performance can be successfully improved.

3 Example of Fuzzy Filter and Its Expermental Results

A fuzzy filter based on three inference rules is proposed to prove the filtering capability of the fuzzy filtering system. Three features of the observation vector are abstracted as the information for the help of filtering work. They are the *rank*, the *bias* and the *gap*. Each feature is corresponding to an input domain of the fuzzy filtering system, that is, χ_1(= domain of *rank*), χ_2(= domain of *bias*) and χ_3(= domain of *gap*). Let $\underline{x}$= $(x_1, x_2, ..., x_N)$ be the observation vector and r_i be the rank of x_i in $(x_1, x_2, ..., x_N)$, that is, x_i = the r_ith rank of the elements x_1, x_2, $\cdots$, and x_N. The second feature is the bias between the mean of the samples and the middle sample $x_{(N/2)}$ (or just say the *bias*). The value of the bias is varied from 0 to $K-1$ where $K-1$ is the maximal value of a pixel. The third feature abstracted is the maximal gap between two successive samples (or just say the *gap*). The largest value of the gap means that the existence of edges or noise. The value of the gap is also variated between 0 and $K-1$ where $K-1$ is the maximal value of the signal.

The three inference rules are given as follows.

Rule 1: If *rank* is *middle-rank* and *bias* is *low-bias*, then output the center sample.

Rule 2: If *rank* is not *middle-rank* and *bias* is *high-bias* and *gap* is *large-gap*, then output the trimmed mean of the observation samples.

Rule 3: If *rank* is not *middle-rank* and *bias* is *high-bias* and *gap* is not *large-gap*, then output the selected rank.

They can be rewritten as

$$R_1 : \text{if } \xi_1 \text{ is } A_1^{(2)} \text{ and } \xi_2 \text{ is } A_2^{(1)} \text{ then } \eta \text{ is } f_1(\underline{x}) \quad (6)$$

where ξ_1 is the domain of *rank*, ξ_2 is the domain of *bias*, $A_1^{(2)}$ represents fuzzy set *middle-rank*, $A_2^{(1)}$ represents fuzzy set *low-bias* and $f_1(\underline{x}) = x_{(N+1)/2}$.

$$R_2 : \text{if } \xi_1 \text{ is not } A_1^{(2)} \text{ and } \xi_2 \text{ is not } A_2^{(1)} \text{ and } \xi_3 \text{ is not } A_3^{(3)} \text{then } \eta \text{ is } f_2(\underline{x}) \quad (7)$$

where ξ_1 is the domain of *rank*, ξ_2 is the domain of *bias*, ξ_3 is the domain of *gap*, $A_1^{(2)}$ represents fuzzy set *middle-rank*, $A_2^{(1)}$ represents fuzzy set *low-bias*, $A_3^{(3)}$represents fuzzy set *large-gap* and $f_2(\underline{x})$ is the trimmed mean of $\underline{x}$.

$$R_3 : \text{if } \xi_1 \text{ is not } A_1^{(2)} \text{ and } \xi_2 \text{ is not } A_2^{(1)} \text{ and } \xi_3 \text{ is } A_3^{(3)} \text{then } \eta \text{ is } f_3(\underline{x}) \quad (8)$$

where ξ_1 is the domain of *rank*, ξ_2 is the domain of *bias*, ξ_3 is the domain of *gap*, $A_1^{(2)}$ represents fuzzy set *middle-rank*, $A_2^{(1)}$ represents fuzzy set *low-bias*, $A_3^{(3)}$represents fuzzy set *large-gap* and $f_3(\underline{x})$ is a $RCRS$ filter.

In the following, the conventional filters are selected for comparison by computer simulations on image restoration to prove the filtering performance of this new proposed fuzzy filters. The median filters (MF) and the center weighted median (CWM) filters and the rank conditioned rank selection ($RCRS$) filters are selected for comparison. The 256×256 Lenna picture with 256 gray levels is used as testing image which is shown in Figure 3.

In Figure 4, it is easy to find that the fuzzy filter with three inference rules is far better than other filters. Even with 20% noise ratio, the fuzzy filters still act very well in comparing with other filters. We believe that the fuzzy filters will be improved when other inference

Figure 3: The original 256 gray-level Lenna picture.

rules for high noise ratio are included in the filtering system.

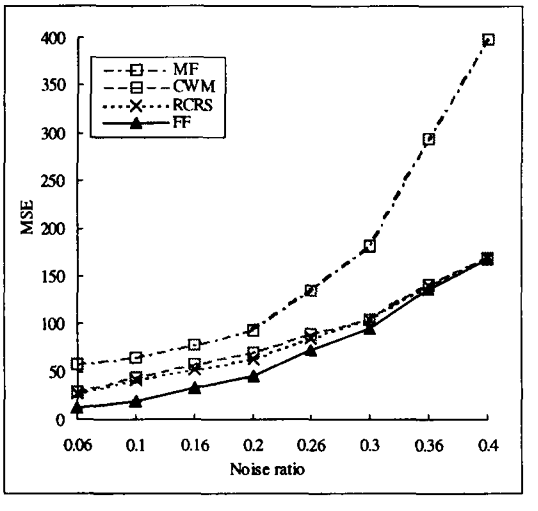

Figure 4: The comparison of nonlinear filters with MSE error criterion.

Finally, the picture corrupted by 20% zero-mean impulsive noise and the filtering result of the fuzzy filters are shown in Figure 5 and Figure 6.

4 Conclusions

In this paper, a fuzzy filtering system has been proposed to aggregate the characteristics of

Figure 5: The picture corrupted by 20% zero-mean impulsive noise.

Figure 6: The filtering result of fuzzy filters.

the conventional filters. In advance, based on this system, a three-inference-rule fuzzy filter has been proposed to improve the filtering capability of the conventional filters. The computer simulations show that the performance of this newly fuzzy filter is far better than the conventional filters for comparison based on the MSE error criteria.

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