IDENTIFICATION OF TIME-VARYING NONLINEAR CHANNELS USING POLYNOMIAL FILTERS

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ABSTRACT

This paper addresses the problem of time-varying (TV) nonlinear system identification. We focus on a class of (almost) periodically TV Volterra series. Such a model is shown to well describe mobile satellite channels which are structured as a time-invariant (TIV) filter cascaded with a TIV zero-memoryless nonlinearity (ZMNL) and a TV linear filter. The nonlinearity distortion is due to the on-board satellite amplifier. The TV filter characterizes fading multipath in mobile environment. A least squares estimate of the TV Volterra kernels with finite memory is first derived for any arbitrary channel input. Then, closed form solutions of the Volterra kernels are derived for symmetrically circular input sequences. The theoretical results are illustrated by simulations.

I INTRODUCTION

Volterra filter is an attractive nonlinear system representation for two reasons: *i*) it is a straightforward generalization of the linear system modelling; *ii*) the parameters to identify are linearly related to the output. Many physical systems are shown to be well modeled by Volterra filters [11]. Time-invariant (TIV) Volterra filter has been intensively studied in the literature (e.g. [6][11]). A little attention has been paid to the more general case of time-varying (TV) Volterra filters.

This paper focuses on (almost) periodically TV Volterra systems, which are characterized by kernels changing with time according to one or multiple periodicities. Such systems are encountered in many applications including rotating machinery and mobile communication systems. The present work concentrates on the second application. Periodically TV Volterra systems are shown to well describe mobile satellite channels (principal component of the third-generation of mobile communication [3]).

Mobile satellite systems are a solution to many technical and economical problems [3]. In this context, satellite Universal Mobile Telecommunications Systems (UMTS) channel is conceived as a multi-application digital mobile system incorporating terrestrial and satellite components. One of the objectives of the European Advanced Communication Technologies and Services (ACTS) project is the modelling and equalization of UMTS channels.

In satellite UMTS channels, three problems occur:
i) Because of the limited availability of band-width, the transmitted signal is severely band-limited in order to allow higher information bit rate. Such a filtering causes significant intersymbole interference (ISI).
ii) Nonlinear distortions are caused by on-board amplifiers such as Travelling Wave Tube (TWT) and Solid State Power (SSP) Amplifiers. Such amplifier devices, used for high speed data transmission, introduce nonlinearity because they usually work near saturation.
iii) Multipath fading is caused by rapid changes of the multipath environment in the earth station to mobile link.
The presence of these distortions leads to a TV nonlinear channel with memory. Thus, for equalization and receiver design, it is important to derive an adequate model for the underlying fading non linear channel.

Modeling and equalization of fixed band-limited satellite channels have been intensively studied in the last two decades. For instance, a Volterra approach has been proposed in [1, 2]; another approach, based on neural networks, has been recently proposed in [5].

The problem of multipath fading is one of the major practical concern in wireless communications. In mobile environment, the multipath is mainly due to the surrounding (e.g. buildings) to the mobile unit. Deterministic as well as stochastic approaches have been proposed to characterize fading multipath linear channels (e.g. [10, 12]).

This paper considers a class of TV nonlinear channels structured as a TIV filter cascaded with a TIV zero-memoryless nonlinearity (ZMNL) and a TV multipath channel. Approximating the ZMNL by a finite-order polynomial, the TV nonlinear channel is shown to be a TV Volterra system. We derive a Least Squares (LS) estimate of the TV Volterra kernels. The LS method does not need particular statistical assumptions on the input signal except certain persistence-of excitation properties. However, the method needs the Volterra kernels to be of finite memory. This restriction is relaxed using circularly symmetric i.i.d. input sequences. Such inputs make the Volterra kernels orthogonal to each other. We therefore derive closed-form solutions of the TV Volterra kernels.

II CHANNEL MODEL

A mobile satellite channel consists of a cascade of a classical satellite channel, describing the satellite to earth basestation links, and a fading multipath channel, describing the earth station to mobile link [3]. In this section, we provide a Volterra series-based model to describe the input-output relationship of the simplified mobile satellite channel given in figure 1.

Let $\{a(k)\}$ be the information symbol sequence. The complex envelope of a linear modulation of $\{a(k)\}$ may be expressed as

$$x(t) = \sum_{k=-\infty}^{\infty} a(k)p(t-kT) \qquad (1)$$

where $p(t)$ is the impulse response of the pulse-shaping filter, T is the data rate.

The modulator output is filtered by a band-limited TIV linear filter. Let $\{h(t)\}$ denote its impulse response. The transmitted signal can then be written as

$$s(t) = Re\left\{\sum_{k=-\infty}^{\infty} a(k)g(t-kT)e^{j\omega_c t}\right\} \triangleq Re\left\{s_b(t)e^{j\omega_c t}\right\} \tag{2}$$

where

$$g(t) = p(t) * h(t)$$

and $*$ denotes convolution.

The nonlinear satellite transponder, which operates at or near saturation, is represented by a ZMNL. The nonlinearity is characterized by amplitude distortion (AM/AM conversion) and phase distortion (AM/PM conversion) [1]. The transponder output can then be written as (using the amplitude and phase representation $s_b(t) = R(t)e^{j\varphi(t)}$)

$$d(t) = Re\left\{f(s_b(t))e^{j\omega_c t}\right\} \triangleq Re\left\{d_b(t)e^{j\omega_c t}\right\} \tag{3}$$

where f is a complex ZMNL. In the sequel, the ZMNL is modeled by an I-th order polynomial, i.e.

$$d_b(t) = \sum_{i=1}^{I} \alpha_i s_b^i(t) \tag{4}$$

where the α_i's are complex coefficients. Thus, the baseband complex envelope of the transponder output is

$$d_b(t) = \sum_{i=1}^{I} \alpha_i \left[\sum_{k=-\infty}^{\infty} a(k)g(t-kT)\right]^i \tag{5}$$

which can be rewritten as

$$d_b(t) = \sum_{i=1}^{I} \alpha_i \sum_{k_1,\dots k_i=-\infty}^{\infty} \left(\prod_{r=1}^{i} a(k_r)\right) \prod_{r=1}^{i} g(t-k_rT) \tag{6}$$

We assume that the bandwidth of the down link filter is large enough to neglect its distortion effects on the transponder output. For simplicity, the same notation ω_c is used for the different carrier frequencies used in the channel.

The multipath fading is usually described as a TV tapped delay line. The time variations are often modeled by a stochastic process [10]. However, for mobile radio channels, it has been shown that the time variations are (almost) periodic (e.g. [12]).

Let L denote the number of delayed versions of $d(t)$ collected at the receiver. The noiseless received signal is

$$r(t) = \sum_{l=1}^{L} \rho_l(t) d(t-\tau_l(t)) \tag{7}$$

where $\rho_l(t)$ and $\tau_l(t)$ are the attenuation factor and the propagation delay for the lth path. Substituting (6) into (7), we obtain

$$r(t) = Re\left\{r_b(t)e^{j\omega_c t}\right\} \tag{8}$$

where $r_b(t)$ is the complex envelope. After sampling, the noisy received discrete-time complex envelope is found to be

$$\begin{aligned} y(n) &= \sum_{i=1}^{I} \alpha_i \sum_{k_1,\dots k_i=-\infty}^{\infty} \left(\prod_{r=1}^{i} a(k_r)\right) \\ &\quad \sum_{l=1}^{L} \rho_{l_n} e^{j\omega_c \tau_{l_n}} \prod_{r=1}^{i} g([n-k_r]T - \tau_{l_n}) + \nu(n) \end{aligned} \tag{9}$$

where $\nu(n)$ is an additive noise including the down-link noise and other disturbances. The complex envelope can be rewritten as

$$\begin{aligned} y(n) &= \sum_{i=1}^{I} \sum_{k_1,\dots k_i=-\infty}^{\infty} \alpha_i \sum_{l=1}^{L} \rho_{l_n} e^{j\omega_c \tau_{l_n}} \prod_{r=1}^{i} g(k_rT - \tau_{l_n}) \\ &\quad \left(\prod_{r=1}^{i} a(n-k_r)\right) + \nu(n) \end{aligned} \tag{10}$$

As in [12], we assume that the τ_{l_n}'s change linearly with time, i.e. $\tau_{l_n} = \lambda_l n + \lambda_l^o$ (which implies that the mobile unit has constant velocity). The time variations of the channel are mainly due to the term $e^{j\omega_c \tau_{l_n}}$. Thus, the time variations of the other terms are neglected in the sequel. Under these assumptions, we obtain

$$y(n) = \sum_{i=1}^{I} \sum_{k_1,\dots k_i} h_i(n;\underline{k}_i) \left(\prod_{r=1}^{i} a(n-k_r)\right) + \nu(n) \tag{11}$$

where $\underline{k}_i = (k_1, \dots, k_i)$, and

$$h_i(n;\underline{k}_i) = \sum_{l=1}^{L} \theta_{i,l}(\underline{k}_i) e^{j\omega_l n} \tag{12}$$

$$\theta_{i,l}(\underline{k}_i) = \alpha_i \left[\rho_{l_n} e^{j\omega_c \lambda_l^o} \prod_{r=1}^{i} g(k_rT - \tau_{l_n})\right] \tag{13}$$

The frequency $\omega_l = \omega_c \lambda_l$, is known as the Doppler frequency associated with the lth path. As mentioned before, the variations of the parameters $\theta_{i,l}(\underline{k}_i)$ w.r.t. n are small compared to that of the exponential $e^{j\omega_l n}$. The $\theta_{i,l}(\underline{k}_i)$'s are assumed complex constants in the following.

The input-output relationship (11) belongs to the class of TV Volterra filtering with factorizable kernels. It is also worth noting that the fading multipath channel model (11) is an extension of the linear model proposed in [12] to nonlinear channels.

III LS ESTIMATION OF VOLTERRA KERNELS

In this section, we assume that the kernels $h_i(n;\underline{k}_i)$ have finite memory length, say q. The discrete-time received signal is then

$$y(n) = \sum_{i=1}^{I} \sum_{k_1,\dots,k_i=0}^{q} h_i(n;\underline{k}_i) \left(\prod_{r=1}^{i} a(n-k_r)\right) + \nu(n) \tag{14}$$

Also, assume that the input sequence satisfies certain persistence-of-excitation conditions [8]. Such conditions guarantee unique determination of the Volterra kernels.

The Volterra kernels can be assumed symmetric without loss of generality, i.e. $h_i(n;\underline{k}_i)$ is left unchanged for all the $i!$ permutations of the indices, $k_1, \dots, k_i$. Thus, the nonredundant regions of the Volterra kernels are

$$F_i = \{\underline{k}_i \,|\, 0 \le k_1 \le \dots \le k_i \le q\}, \quad i = 1, \dots, I$$

Let $P(\underline{k}_i)$ denote the number of distinguishable permutations of $(k_1, .., k_i)$, which can be expressed as

$$P(\underline{k}_i) = \frac{i!}{\prod_{r=1}^{i}\left(\sum_{j=r}^{i} \delta(k_r - k_j)\right)} \tag{15}$$

For example, $P(\underline{k}_i) = i!$ when $k_1 \neq ... \neq k_i$. and $P(\underline{k}_i) = 1$ when $k_1 = ... = k_i$.

The signal model (14) can be rewritten as

$$y(n) = \sum_{i=1}^{I} \sum_{\underline{k}_i \in \Gamma_i} h_i(n; \underline{k}_i) P(\underline{k}_i) \left(\prod_{r=1}^{i} a(n - k_r) \right) + \nu(n) \tag{16}$$

Let us define the following notations

$$\begin{aligned}
\boldsymbol{\theta}_i(\underline{k}_i) &\triangleq [\theta_{i,1}(\underline{k}_i), .., \theta_{i,L}(\underline{k}_i)] \\
\boldsymbol{\theta}_i &\triangleq [\boldsymbol{\theta}_i(0, .., 0), \boldsymbol{\theta}_i(0, .., q), ..., \boldsymbol{\theta}_i(q-1, q, ..., q), \\
&\qquad \boldsymbol{\theta}_i(q, q, ..., q)] \\
\boldsymbol{\theta} &\triangleq [\boldsymbol{\theta}_1, ..., \boldsymbol{\theta}_I]^T \\
\boldsymbol{\omega} &\triangleq [\omega_1, ..., \omega_L]^T
\end{aligned}$$

and

$$\begin{aligned}
a_i(n; \underline{k}_i) &\triangleq P(\underline{k}_i) \prod_{r=1}^{i} a(n - k_r) \\
\mathbf{a}_i(n) &\triangleq [a_i(n; 0, .., 0), ..., a_i(n; q-1, q, ..., q), \\
&\qquad a_i(n; q, q, ..., q)] \\
\mathbf{a}(n) &\triangleq [\mathbf{a}_1(n), ..., \mathbf{a}_I(n)] \\
\boldsymbol{\phi}(\boldsymbol{\omega}; n) &\triangleq [e^{j\omega_1 n}, ..., e^{j\omega_L n}]^T
\end{aligned}$$

The signal model (14) can then be expressed as

$$y(n) = \sum_{i=1}^{I} \sum_{\underline{k}_i \in \Gamma} h_i(n; \underline{k}_i) P(\underline{k}_i) \left(\prod_{r=1}^{i} a(n - k_r) \right) + \nu(n) \tag{17}$$

which can be rewritten as

$$y(n) = (\mathbf{a}(n) \otimes \boldsymbol{\phi}(\boldsymbol{\omega}; n)) \boldsymbol{\theta} + \nu(n) \tag{18}$$

where $\otimes$ denotes the Kronecker product operator.

Collecting N measurements $\mathbf{y} = (y(0), ..., y(N-1))^T$ of the received signal yields the following matrix formulation

$$\mathbf{y} = \mathbf{A}(\boldsymbol{\omega}) \boldsymbol{\theta} + \boldsymbol{\nu} \tag{19}$$

where

$$\mathbf{A}(\boldsymbol{\omega}) = \begin{bmatrix} \mathbf{a}(0) \otimes \boldsymbol{\phi}(\boldsymbol{\omega}; 0) \\ \mathbf{a}(1) \otimes \boldsymbol{\phi}(\boldsymbol{\omega}; 1) \\ \vdots \\ \mathbf{a}(N-1) \otimes \boldsymbol{\phi}(\boldsymbol{\omega}; N-1) \end{bmatrix}$$

and $\boldsymbol{\nu} = (\nu(0), ..., \nu(N-1))^T$. The parameter vectors to estimate are then $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$.

Assuming that the additive noise is white, the LS estimates of $\boldsymbol{\theta}$ and $\boldsymbol{\omega}$ are given by

$$(\widehat{\boldsymbol{\theta}}, \widehat{\boldsymbol{\omega}}) = \arg\min_{\boldsymbol{\theta}, \boldsymbol{\omega}} J_1(\mathbf{y}; \boldsymbol{\theta}, \boldsymbol{\omega}) \tag{20}$$

where

$$J_1(\mathbf{y}; \boldsymbol{\theta}, \boldsymbol{\omega}) = (\mathbf{y} - \mathbf{A}(\boldsymbol{\omega})\boldsymbol{\theta})^H (\mathbf{y} - \mathbf{A}(\boldsymbol{\omega})\boldsymbol{\theta}) \tag{21}$$

If the Doppler frequency vector $\boldsymbol{\omega}$ is known, the LS estimate of $\boldsymbol{\theta}$ is given by

$$\widehat{\boldsymbol{\theta}} = \left(\mathbf{A}(\boldsymbol{\omega})^H \mathbf{A}(\boldsymbol{\omega})\right)^{\dagger} \mathbf{A}(\boldsymbol{\omega})^H \mathbf{y} \tag{22}$$

where † stands for the pseudoinverse. In practice, $\boldsymbol{\omega}$ is unknown. Substituting (22) into (21), we have now to minimize the following criterion

$$J(\mathbf{y}; \boldsymbol{\omega}) = \mathbf{y}^H (I - P_{\mathbf{A}(\boldsymbol{\omega})}) \mathbf{y} \tag{23}$$

where $P_{\mathbf{A}(\boldsymbol{\omega})}$ is the projection matrix on the signal subspace (if $\boldsymbol{\omega}$ is the true Doppler frequency vector)

$$P_{\mathbf{A}(\boldsymbol{\omega})} = \mathbf{A}(\boldsymbol{\omega}) \left(\mathbf{A}(\boldsymbol{\omega})^H \mathbf{A}(\boldsymbol{\omega})\right)^{\dagger} \mathbf{A}(\boldsymbol{\omega})^H \tag{24}$$

Since $J_1(\mathbf{y}; \boldsymbol{\omega})$ is nonlinear w.r.t. $\boldsymbol{\omega}$, we cannot derive an analytical solution for the Doppler frequencies. Thus, we resort to numerical optimization techniques, e.g.

$$\widehat{\boldsymbol{\omega}}^{(i+1)} = \widehat{\boldsymbol{\omega}}^{(i)} - \mu \nabla_{\boldsymbol{\omega}} J(\mathbf{y}; \boldsymbol{\omega}) \tag{25}$$

To ensure rapid convergence of the algorithm, we need an initialization $\widehat{\boldsymbol{\omega}}^{(0)}$ of the Doppler frequencies. The next section can be used for this matter. The next section also derives closed form solutions of the Volterra kernels when the input is a circular Gaussian independent sequence.

IV VOLTERRA IDENTIFICATION FOR CIRCULARLY SYMMETRIC INPUTS

In the previous section, apart from the persistence-of-excitation property, we made no additional statistical assumption on the input sequence. However, the Volterra kernels were assumed to have finite memory lengths. This condition can be relaxed if special inputs are used in the identification procedure. We then reconsider the Volterra system in (11). The non-redundant regions of the Volterra kernels are redefined as

$$\Gamma_i = \{\underline{k}_i \mid -\infty \leq k_1 \leq ... \leq k_i \leq \infty\}, \quad i = 1, ..., I$$

The idea motivating the use of special inputs is to make the Volterra kernels orthogonal. Real stationary Gaussian input has been used in [7]. In this, case, the solutions are rather complicated except for second-order Volterra systems. An alternative solution using cyclostationary inputs has been proposed in [4]. Volterra system identification using PSK sequences as input has been considered in [13]. In this paper, we show that closed form solutions can be obtained for any circularly symmetric independent input sequences.

Let a be a complex random variable and let ρ and ϕ denote its modulus and its phase (modulo 2π). a is said circular if for any β, a and $a\exp(j\beta)$ have the same distribution [9]. This implies that ρ and ϕ are independent and that ϕ is uniformly distributed over $[0, 2\pi)$. In the Gaussian case, c has a Rayleigh distribution. Let the input sequence $\{a(n)\}$ be formed by independent realizations of a.

Let $\underline{\tau}_r = (\tau_1, ..., \tau_r)$, $1 \leq r \leq I$. Consider the cross-correlation between the output $y(n)$ and conjugated and lagged copies of the input $a(n)$:

$$m_{ya...a}(n; \underline{\tau}_r) \triangleq E\left\{ y(n) \prod_{s=1}^{r} a^*(n - \tau_s) \right\} \tag{26}$$

where $\underline{\tau}_r = (\tau_1, ..., \tau_r)$, $\tau_1 \leq ... \leq \tau_r$, $1 \leq r \leq I$. Rewritting (11) as in (14), we obtain

$$m_{ya...a}(n; \underline{\tau}_r) = \sum_{i=1}^{I} \sum_{\underline{k}_i \in \Gamma} h_i(n; \underline{k}_i) P(\underline{k}_i) m_{a...a}(\underline{k}_i; \underline{\tau}_r) \tag{27}$$

where

$$m_{a\ldots a}(\underline{k}_i;\underline{\tau}_r) \triangleq E\left\{\prod_{s=1}^{i} a(n-k_s)\prod_{s=1}^{r} a^*(n-\tau_s)\right\}$$

The circular symmetry implies the following property [9]

$$m_{a\ldots a}(\underline{k}_i;\underline{\tau}_r) = 0 \qquad (28)$$

when $i \neq r$. This ensures the orthogonality of Volterra kernels having different order of nonlinearity. Thus, we can estimate separately the linear kernel, the quadratic kernel, the cubic kernels and so on. We can then drop $\sum_{i=1}^{I}$ in (27) and obtain the following equations

$$m_{ya\ldots a}(n;\underline{\tau}_i) = \sum_{\underline{k}_i\in F_i} h_i(n;\underline{k}_i)P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{\tau}_i) \qquad (29)$$

The $2i$th moment $m_{a\ldots a}(\underline{k}_i;\underline{\tau}_i)$ can be written as

$$\begin{aligned} m_{a\ldots a}(\underline{k}_i;\underline{\tau}_i) &= E\left\{\prod_{s=1}^{i}\rho(n-k_s)\rho(n-\tau_s)\right. \qquad (30)\\ &\qquad \left.\exp j\sum_{s=1}^{i}(\phi(n-k_s)-\phi(n-\tau_s))\right\} \end{aligned}$$

Using the independence of the modulus and the phase of circular random variables, we obtain

$$\begin{aligned} m_{a\ldots a}(\underline{k}_i;\underline{\tau}_i) &= E\left\{\prod_{s=1}^{i}\rho(n-k_s)\rho(n-\tau_s)\right\} \qquad (31)\\ &\qquad E\left\{\exp j\sum_{s=1}^{i}(\phi(n-k_s)-\phi(n-\tau_s))\right\} \end{aligned}$$

Note that the vectors $\underline{k}_i$ and $\underline{\tau}_i$ are such that $k_1 \leq k_2 \leq \ldots \leq k_i$ and $\tau_1 \leq \tau_2 \leq \ldots \leq \tau_i$. Moreover, the $\phi(n)$'s are independent and $E\{\exp j\phi(n)\} = 0$. This yields

$$m_{a\ldots a}(\underline{k}_i;\underline{\tau}_i) = 0 \quad \text{if } \underline{k}_i \neq \underline{\tau}_i \qquad (32)$$

We can then drop $\sum_{\underline{k}_i\in F_i}$ in (29):

$$m_{ya\ldots a}(n;\underline{k}_i) = h_i(n;\underline{k}_i)P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i) \qquad (33)$$

Thus, we infer the following closed form solutions

$$h_i(n;\underline{k}_i) = \frac{m_{ya\ldots a}(n;\underline{k}_i)}{P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i)}, \qquad k_i \in F_i \qquad (34)$$

where $i = 1,\ldots,I$ and

$$m_{a\ldots a}(\underline{k}_i;\underline{k}_i) = E\left\{\prod_{s=1}^{i}|a(n-k_s)|^2\right\}$$

For Gaussian circular inputs, we obtain

$$m_{a\ldots a}(\underline{k}_i;\underline{k}_i) = \frac{m_{a\ldots a}(\underline{0}_i;\underline{0}_i)}{P(\underline{k}_i)} \qquad (35)$$

where $m_{a\ldots a}(\underline{0}_i;\underline{0}_i)$ is the $2i$th order absolute moment of $\{a(n)\}$:

$$\begin{aligned} m_{a\ldots a}(\underline{0}_i;\underline{0}_i) &\triangleq E\left\{|a(n)|^{2i}\right\}\\ &= i!\,\sigma^{2i} \end{aligned}$$

where σ^2 denotes the variance of $a(n)$. Thus, in this case, the Volterra kernels are given by

$$h_i(n;\underline{k}_i) = \frac{m_{ya\ldots a}(n;\underline{k}_i)}{i!\,\sigma^{2i}}, \qquad \underline{k}_i \in F_i,\ i \geq 1 \qquad (36)$$

The Volterra kernel estimates are derived by replacing the theoretical moments $m_{ya\ldots a}(n;\underline{k}_i)$ by their estimates $\widehat{m}_{ya\ldots a}(n;\underline{k}_i)$ in (34):

$$\widehat{h}_i(n;\underline{k}_i) = \frac{\widehat{m}_{ya\ldots a}(n;\underline{k}_i)}{P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i)}, \qquad k_i \in F_i,\ i \geq 1 \qquad (37)$$

If the Volterra kernels are TIV i.e. $m_{ya\ldots a}(n;\underline{k}_i) = m_{ya\ldots a}(\underline{k}_i)$, consistent estimate can be obtained from a single record of input-output data:

$$\widehat{m}_{ya\ldots a}(\underline{k}_i) = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\prod_{s=1}^{i} a^*(n-k_s) \qquad (38)$$

For the (almost) periodically TV Volterra system (11), we consider the following cyclic cross-correlations

$$M_{ya\ldots a}(\lambda;\underline{k}_i) \triangleq \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N} m_{ya\ldots a}(n;\underline{k}_i)e^{-j\lambda n} \qquad (39)$$

According to (12), we get

$$M_{ya\ldots a}(\lambda;\underline{k}_i) = P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i)\sum_{l=1}^{L}\theta_{i,l}(\underline{k}_i)\delta(\lambda-\omega_l) \qquad (40)$$

Thus, the statistic $|M_{ya\ldots a}(\lambda;\underline{k}_i)|$ peaks at the Doppler frequencies ω_l, $l = 1,\ldots L$, provided $\theta_{i,l}(\underline{k}_i) \neq 0$. This also allows the determination of L, the number of paths. Once the ω_l's are retrieved, the $\theta_{i,l}(\underline{k}_i)$ can be obtained as

$$\theta_{i,l}(\underline{k}_i) = \frac{M_{ya\ldots a}(\omega_l;\underline{k}_i)}{P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i)} \qquad (41)$$

From a single record of input-output data, the cyclic statistic $M_{ya\ldots a}(\lambda;\underline{k}_i)$ can be consistently estimated by

$$\widehat{M}_{ya\ldots a}(\lambda;\underline{k}_i) = \frac{1}{N}\sum_{n=0}^{N-1} y(n)\prod_{r=1}^{i} a^*(n-k_r)e^{-j\lambda n} \qquad (42)$$

A consistent estimation of the Volterra kernels is then obtained as

$$\widehat{h}_i(n;\underline{k}_i) = \frac{\sum_{l=1}^{L}\widehat{M}_{ya\ldots a}(\widehat{\omega}_l;\underline{k}_i)e^{j\widehat{\omega}_l n}}{P(\underline{k}_i)m_{a\ldots a}(\underline{k}_i;\underline{k}_i)} \qquad (43)$$

where the estimate $\widehat{\omega}_l$'s are obtained using the peak picking technique on the statistic $\widehat{M}_{ya\ldots a}(.;.)$.

V SIMULATION RESULTS

In this paper, we present simulations for the LS estimation developed in section III. Detailed study of the method proposed in section IV will be presented elsewhere. However, the initialization of the Dopller frequencies in the LS algorithm is provided using the cyclic cross correlations developed in section IV.

Consider a linear-quadratic Volterra system. The input sequence is Gaussian circular and white. The ZMNL coefficients are $\alpha_1 = 1$ and $\alpha_2 = 0.5 + 0.2i$. We consider a multipath environment composed by a direct path i.e. $\omega_1 = 0$, and a reflected path with Doppler frequency $\omega_2 = 2\pi \times 0.05$. The system memory is set to $q = 10$. The signal-to-noise ratio is $SNR = 10$dB.

Figure 2 represents the average of the second-order cyclic cross correlations over $k_1 = 0, ..., q$ from a single record of input-output data of length $N = 128$. The MSEs of the linear and quadratic kernels LS estimates are computed using Monte-Carlo experiments. Figure 3 displays the averages of the relative MSEs of the linear and quadratic kernel estimations.

VI CONCLUSIONS

This paper addressed (almost) periodically TV Volterra systems. Such systems are shown to well describe mobile satellite channels. We first derived a least squares estimate of the TV Volterra kernels. Then, we focused on circularly symmetric inputs. Circularity and independence of the input sequence make the Volterra kernels orthogonal to each other. We therefore derived closed-form solutions for the TV Volterra kernels.

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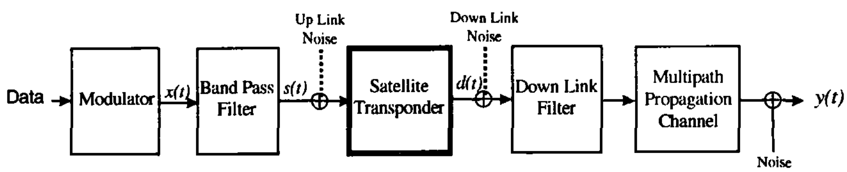

Fig. 1 : mobile satellite channel

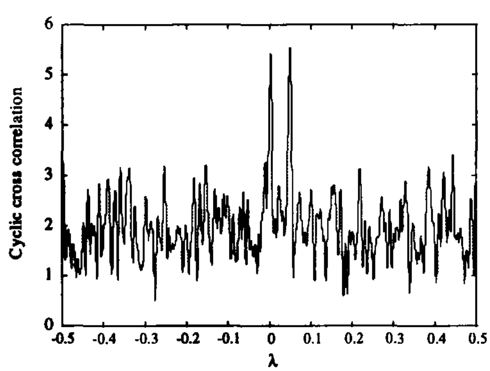

Fig. 2. Initial Estimation of the Doppler frequencies. $N = 128$

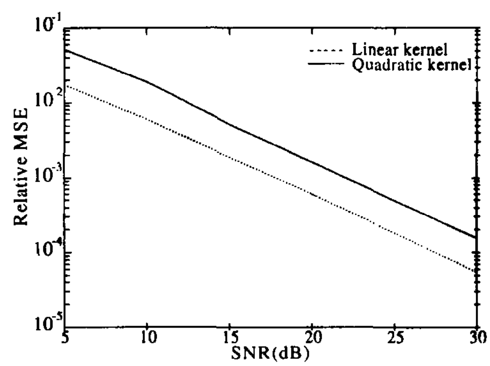

Fig. 3. MSE on the LS estimation. $N = 128$.