Data-Dependent Linear Combination of Weighted Order Statistics (DD-LWOS) Filtering Based on Local Statistics

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ABSTRACT

Nonlinear filters which are utilized rank-order information and temporal-order information, have many proposed, in order to restore nonstationary signals which are corrupted by additive noise. LWOS (Linear Combination of Weighted Order Statistics) filters [1] which also utilized two informations , and have properties of efficient impulsive and non-impulsive noise attenuation and sufficiently details and edges preservation.

In this paper, we propose a data-dependent LWOS filter whose coefficients change based on local statistics.

1. INTRODUCTION

It is well known that median filters are very useful for image restoration. The success of median filters is based on two properties: edge preservation and efficient impulsive noise attenuation. However, the median filter is not a perfect filtering operation. It may remove important image details and not sufficiently attenuate non-impulsive noise. The main reason is that the median filter uses only rank-order information of the input data within the filter window, and discards its original temporal-order information. In order to utilize both rank- and temporal-order information of input data, several classes of rank order based filters have been developed in recent years, such as Ll filters[4], Weighted median filters[2], FIR-WOS(weighted order statistics) hybrid filters[3].

J. Song and Y. H. Lee have also proposed a filter whose structure provide a unified framework for representing both linear FIR and median type nonlinear filters called linear combination of weighted order statistics (LWOS) filters[1]. The LWOS filter is combination of L-filters and WOS filters. In this paper, we propose adaptive LWOS filters based on local statistics called data-dependent LWOS (DD-LWOS) filters. The DD-LWOS filter is a combination of the data-dependent α-trimmed mean filter[5] and the data-dependent weighted median filter[6]. They are useful in image processing because for typical image each part differs sufficiently from other parts. Several design examples are presented showing the good performance of the proposed filter.

2. DD-LWOS FILTERS

2-1 LWOS FILTERS

The output of LWOS filters is given by

$$y(n)=\sum_{j=1}^{J} l_j \cdot X_{(j/J)}(n) \tag{1}$$

where l_i is a coefficient of L-filters and $X_{(j/J)}(n)$ is j th largest samples of in a row vector $X(n)$ defined by

$$X(n)=\left[w_{-N}\Diamond x(n-N),\cdots,w_0\Diamond x(n),\cdots,w_N\Diamond x(n+N)\right] \tag{2}$$

$w\Diamond x$ denotes the repeating operation, i.e.,

$$w\Diamond x=\overbrace{x,\cdots x}^{wtimes} \tag{3}$$

J is a degree of vector $X(n)$ as

$$J=\sum_{i=-N}^{N} w_i \tag{4}.$$

The class of LWOS filters encompasses WOS and L-filters.

Examples:
Consider a filter windows length 5 LWOS filter with weights as

$[w_{-2}, w_{-1}, w_0, w_1, w_2] = [1, 1, 3, 1, 1]$
$[l_1, l_2, l_3, l_4, l_5, l_6, l_7] = [0.1, 0.1, 0.1, 0.4, 0.1, 0.1, 0.1]$

and an input vector

$[x_{-2}, x_{-1}, x_0, x_1, x_2] = [2, 3, 7, 6, 8]$.

Then $X(n)$ given by

$X(n) = [2, 3, 7, 7, 7, 6, 8]$.

The output of LWOS filter calculated by

$$y = \sum_{j=1}^{7} l_j \cdot X_{(j/7)} = 0.1 \cdot 2 + 0.1 \cdot 3 + 0.1 \cdot 6 + (0.4 + 0.1 + 0.1) \cdot 7 + 0.1 \cdot 8 = 6.1$$

2-2 DD-LWOS FILTERS

Data-dependent non-linear filters useful in image processing because for a typical images are non-stationary. Several data-dependent nonlinear filters, such as data-dependent WM filters[6],[7], and data-dependent L-filters[5], are proposed. In this paper we propose a novel data-dependent filter called DD-LWOS filter. The DD-LWOS filter is a combination of the data-dependent α-trimmed mean filter[5] and the data-dependent weighted median (WM) filter[6]. The output of DD-LWOS filters is defined by

$$y(n) = \frac{\sum_{j=\alpha(n)J(n)+1}^{J(n)-\alpha(n)J(n)} \hat{X}_{(j/J(n))}(n)}{J(n) \cdot (1 - 2\alpha(n))} \tag{5}$$

where $\hat{X}_{(j/J(n))}(n)$ is j th largest samples of in a row vector $\hat{X}(n)$ defined by

$$\hat{X}(n) = \{w_{-N}(n) \Diamond x(n-N), \cdots, w_0(n) \Diamond x(n), \cdots, w_N(n) \Diamond x(n+N)\} \tag{6}$$

The number of elements of vector $\hat{X}(n)$ is given by

$$J(n) = \sum_{i=-N}^{N} w_i(n) \tag{7}$$

Thus, $J(n)$ is changed at each point "n". Equation (5) shows the definition of data-dependent α-trimmed mean filters. The parameter "$\alpha(n) \cdot J(n)$" is defined as

$$\alpha(n) \cdot J(n) = \lceil K(n) \cdot (J(n) - 1)/2 \rceil \tag{8}$$

where the $\lceil x \rceil$ is denoted the integer number which is rounded x. Equation (6) shows the definition of data-dependent WM filter. The weights $w_k(n)$ is given by

$$w_k(n) = \lceil W_T \cdot K(n) \cdot D_k \cdot E_k \rceil + 1 \tag{9}$$

$$D_k = \left(1 - \frac{d_k}{L}\right) \tag{10}$$

$$E_k = \begin{cases} 1 & : \text{if } e_k \le \mu \cdot \sigma_n \\ 0 & : \text{otherwise} \end{cases} \tag{11}$$

where $e_k = |x(n+k) - x(n)|$ and σ_n is a standard deviation of the additive white noise. d_k is the distance between $x(n)$ and $x(n+k)$. W_T is an integer. $K(n)$ in Eq. (8)(9), is coefficient that is determined by the local statistics, and is calculate by

$$K(n) = \frac{\sigma^2(n)}{\sigma^2(n) + \sigma_n^{\ 2}} \tag{12}$$

where

$$\sigma^2(n) = \begin{cases} \text{var}(n) - \sigma_n^{\ 2} & : \text{if } \text{var}(n) \ge \sigma_n^{\ 2} \\ 0 & : \text{otherwise} \end{cases} \tag{13}$$

Local variance $var(n)$ and local average $ave(n)$ are given by

$$\text{var}(n) = \left\{ \sum_{i=-N}^{N} (x(n+i) - ave(n))^2 \right\} / 2N + 1 \tag{14}$$

$$ave(n) = \left(\sum_{i=-N}^{N} x(n+i) \right) / 2N + 1 \tag{15}$$

Coefficient $K(n)$ lies between 0 to1 when the local variance changes. In areas with low signal to ratio (i.e., smooth areas corrupted by non-impulsive noise), since $K(n) \approx 0$ then $w_k(n)$ is close to one and "$\alpha(n) \cdot J(n)$" is close to zero. Thus, DD-LWOS filters almost equal to mean filters. On the other hand, in areas with high signal to noise ratio, "$\alpha(n) \cdot J(n)$" is close to $(J(n)-1)/2$ then, DD-LWOS filters equal to WM filters with high center weight

$w_0(n)$ which can preserve signal details. DD-LWOS filters can remove non-impulsive noises while preserving signal details.

2-3 DD-LWOS FILTER WITH IMPULSE NOISE INFORMATION

When the point "*n*" is corrupted by impulsive noise, DD-LWOS filter tend to preserve this impulse noise. Because, the information E_k is affected by impulsive noise. Thus, we propose DD-LWOS filter with new information $\hat{E}_k$ which is derived by addition the impulse noise information to the information E_k .The new different information $\hat{E}_k$ is defined by

$$\hat{E}_k = \begin{cases} 1: & if\ |x(n+k) - z(n)| \geq \mu \cdot \sigma_n \\ 0: & otherwise \end{cases} \qquad (16)$$

where

$$z(n) = x(n) + I(n) \cdot \{X_{med}(n) - x(n)\} \qquad (17).$$

Impulse noise information *I*(*n*) is given by

$$I(n) = \begin{cases} 1: & if\ |X_{med}(n) - x(n)| \geq \varepsilon \\ 0: & otherwise \end{cases} \qquad (18)$$

where $X_{med}(n)$ is the output of median filter with small window (e.g.3×3) and ε is threshold parameter. The weights of DD-LWOS filter with the information $\hat{E}_k$ is given by Eq. (19), instead of Eq. (9).

$$w_k(n) = \lceil W_T \cdot K(n) \cdot D_k \cdot \hat{E}_k \rceil + 1 \qquad (19).$$

We call this filter DD-LWOS 2 filter. From Eq.(17), when the point "*n*" is corrupted by impulsive noise, $z(n)=X_{med}(n)$. Thus, the different information $\hat{E}_k$ is calculated by using $X_{med}(n)$. On the other hand, the point "*n*" is not corrupted by impulsive noise, different information is given by *x*(*n*). Therefore, the impulsive noise affection is avoided in the DD-LWOS 2 filter. DD-LWOS 2 filter gets more better performance than DD-LOWS filter when input image corrupted by additive noise which includes impulsive noise components.

3.EXAMPLES

The performance of DD-LWOS filter and DD-LWOS 2 filter in image restoration is demonstrated and compared to the data-dependent WM filter, adaptive center weighted median filter[7], Median filter and Wiener filter. A 5×5 window is used for all filters.

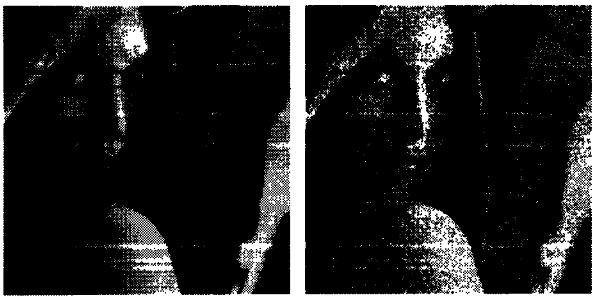
An 8-bits test image "Lena" which contain 256×256 pixels, is used in the simulations. First, the original image corrupted by Gaussian noise (zero mean and standard deviation σ_n =10,20). Second, the Gaussian noise(σ_n =20) corrupted by image further mixed with bipolar impulsive noise with probabilities 0.04 and 0.08. MESs measure the errors between the filtered images and the original image and the measured results are listed in Table 1 and Table 2. And image results(σ_n =20, σ_n =20+0.04) are showed Fig.1 and Fig.2. It is easy to see that DD-LWOS filters are better than data-dependent WM filter. In mixed noise case, DD-LWOS 2 is shown as best results.

Table 1 The measured MSEs for Gaussian noisy test image(Lena).

filter type	σ_n=10	σ_n=20
Identity	92.4	388.6
DD-LWOS	39.3	94.5
AMW	41.6	104
ACMW	49.3	122.4
Wiener	53.3	119.1
Median	151.6	180

Table 2 The measured MSEs for mixed noisy test image(Lena).

filter type	σ_n=20+4%	σ_n=20+8%
Identity	1136.7	1797.6
DD-LWOS	123.7	136.5
DD-LWOS 2	111.8	125.7
AMW	125.4	143.6
ACMW	150.1	159.8
Wiener	201.6	254.5
Median	186.2	192.1



(a) Original image (b) Noisy image

(c) Wiener filter (d) DD-LWOS filter

Fig.1 : Image results (Gaussian noise)

(a) Original image (b) Noisy image

(c) Median filter (d) DD-LWOS filter

Fig.2 : Image results (Mixed noise)

CONCLUTIONS

In this paper we proposed the DD-LWOS filter based on local statistics. Since the proposed filter use both rank-order and temporal-order information of local data, it was showed good performances of smoothing noise not only Gaussian type but also impulse type and preserving details and edges.

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