ROBUST FILTERING

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1. SUMMARY

Robust estimation is not a new subject of study. Indeed, it is a major problem in statistics and signal processing. For example, Huber notes in 1964 [6] that when the true distribution deviates, even mildy, from an assumed Gaussian, the variance of the sample mean may explode. This motivated his pursuit of a general theory of robust estimation. As stated in the survey talk [8], "robust signal processing techniques are techniques with good performance under any nominal conditions and acceptable performance for signal and noise conditions other than the nominal which can range over the whole of allowable classes of possible characteristics". A common approach to robust estimation is to make use of minimax techniques, which offer optimal worst-case performance.

This talk considers filtering problems where it is desired to estimate a function of a signal state using the information available in certain noisy observations of the signal. When the least squares criterion is used and when we assume the plant is completely and accurately described by an assumed model, then the optimal estimator is the conditional mean, computed using the assumed model. In view of the problems which can be encountered when the plant deviates from the assumed model, we consider, following Huber, the use of other error criteria. The risk-sensitive criterion is one such alternative, and in the context of robust control theory, it is known to enjoy robustness characteristics by virtue of its close connection with the H_∞ criterion. For this reason, several authors have considered risk-sensitive filters. However, no robustness results for risk-sensitive filters have appeared to date in the literature, to our knowledge.

This talk gives a precise meaning to the robustness of risk-sensitive filters. We assume that the true probabilistic model is fixed but unknown, and that the estimation (or filtering) procedure makes use of a fixed nominal model. It is shown that risk-sensitive estimators (including filters) enjoy an error bound which is the sum of two terms, the first of which coincides with an upper bound on the error one would obtain if one knew exactly the underlying probability model, while the second term is a measure of the distance between the true and design probability models. The first term quantifies "good performance" under nominal conditions, and the second term quantifies the "acceptable performance" under non-nominal conditions. Further, the second term plays a major role in determining the class of permissible variations from nominal.

Suppose we are given a measurable space $(\Omega, \mathcal{F})$ and random variables X, Y. Here, Y represents the observations (measurements) and $\phi = \phi(X)$ is a (real-valued) function of X to be estimated by a random variable $\hat{\phi}$ which is $\mathcal{Y} = \sigma\{Y\}$ measurable (denoted $\hat{\phi} \in \mathcal{Y}$). Further, let us suppose that we do not know the underlying probability distribution; however, we assume that the "true" distribution P_{α_0} belongs to a family of probability measures $\{P_\alpha\}_{\alpha \in A}$, where A is an arbitrary index set (not necessarily a subset of a finite dimensional space). Since we do not know the value of the parameter α_0, we use a design value α_d for the purpose of constructing the estimator $\hat{\phi}$. It is not necessary that the design parameter α_d belong to A, and indeed the results presented in this talk are *nonparametric*; the α's are used simply as labels.

The *minimum risk-sensitive estimator* (MRSE) cost function is defined by

$$f_{rs}(\hat{\phi}) = \mathbf{E}_{\alpha_d}[\exp(\mu\rho(\phi - \hat{\phi}))] \tag{1}$$

where ρ is a non-negative strictly convex function, $\mu > 0$ is a risk parameter, and the MRSE can be defined

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uniquely as the equivalence class of $\mathcal{Y}$-measurable random functions achieving the minimum

$$\hat{\phi}^*_{rs} = \underset{\hat{\phi} \in \mathcal{Y}}{\mathrm{argmin}} f_{rs}(\hat{\phi}) \qquad (2)$$

In order to write down the error bound below, we need to assume that the true measure is absolutely continuous with respect to the design measure: $P_{\alpha_0} << P_{\alpha_d}$, since otherwise the bound is vacuous. The proof of the bound follows from the well-known duality between free energy and relative entropy.

Theorem 1.1 *Assume there exists a Y-measurable random function $\hat{\phi}$ such that $f_{rs}(\hat{\phi}) < \infty$. Then the MRSE defined by (2) exists, and enjoys the following error bound:*

$$\mathbf{E}_{\alpha_0}[\rho(\phi - \hat{\phi}^*_{rs})] \leq \tfrac{1}{\mu}\{\log f_{rs}(\hat{\phi}^*_{rs}) + R(P_{\alpha_0}|P_{\alpha_d})\}, \quad (3)$$

where $R(P_{\alpha_0}|P_{\alpha_d})$ is the relative entropy.

Signal processing problems where it is reasonable to assume "absolutely continuous uncertainty" are common, and many results in the literature fall into this category (e.g. linear-Gaussian signal models with uncertainty in the state transition matrix). In the following sections we study nonlinear filtering problems with uncertainty and present MRSE filters and the corresponding error bounds.

The remainder of the talk considers filtering problems in both the risk-sensitive and H_∞ contexts.

Full details are available in the paper [1].

2. REFERENCES

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