A BAYESIAN FRAMEWORK FOR NONLINEAR DIFFRACTION TOMOGRAPHY

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ABSTRACT

The Bayesian approach has been proven to give a common estimation structure to existing image reconstruction and restoration methods [1]. The goal of this paper is to investigate diffraction tomography in this framework. A regularized solution to this nonlinear inverse problem is defined as the maximum *a posteriori* estimate, introducing prior information on the object to be reconstructed. Two equivalent formulations of this definition are proposed which lead to solution of a constrained or an unconstrained optimization problem. From this point of view, we propose a classification of existing methods for solving this problem and new orientations to compute the defined solution.

Introduction

Diffraction tomography (DT) consists in constructing an image representing the spatial variation of some physical properties of an inhomogeneous object (such as dielectric permittivity and conductivity for electromagnetic waves), from a data set of field scattered by this object. In addition to its ill-posedness, the characteristic of this problem is given by its nonlinear object/data relation. The objectives of this paper are *i)* to define a regularized solution to this nonlinear inverse problem in the Bayesian estimation framework; *ii)* to propose a classification of some of the existing methods as algorithms to compute the defined solution; *iii)* to propose new orientations and algorithms for this computation in each distinguished class.

After a brief presentation of the direct problem model, we define a regularized solution in the Bayesian estimation framework. The maximum *a posteriori* estimate is considered, so the solution's computation requires to solve an optimization problem.

Then a classification of existing methods to solve the DT problem is proposed. We have distinguished three classes of methods: the first consider successive linearizations of the direct model, the second define the solution as the minimum of a joint criterion depending both on the object and on the field in the object, the third minimize a criterion which only depends on the object. For each of these classes, we briefly describe the methods' principles and we propose new orientations to compute the defined regularized solution.

1. PROBLEM STATEMENT

We consider an inhomogeneous 2-D object, embedded in a known background medium, illuminated with a pure harmonic Transverse Magnetic plane wave. The object is characterized by its complex contrast function $x(\boldsymbol{r}), \boldsymbol{r} \in \mathbb{R}^2$ which is directly related to the dielectric permittivity and the conductivity of the object. The direct scattering problem is modeled by the coupled integral equations:

$$y(\boldsymbol{r}_i) = \iint_{D_\mathrm{O}} \mathcal{G}(\boldsymbol{r}_i, \boldsymbol{r}')\ x(\boldsymbol{r}')\ \phi(\boldsymbol{r}')\ \mathrm{d}\boldsymbol{r}',\ \ \boldsymbol{r}_i \in D_\mathrm{M},$$

$$\phi(\boldsymbol{r}) = \phi_0(\boldsymbol{r}) + \iint_{D_\mathrm{O}} \mathcal{G}(\boldsymbol{r}, \boldsymbol{r}')\ x(\boldsymbol{r}')\ \phi(\boldsymbol{r}')\ \mathrm{d}\boldsymbol{r}',\ \ \boldsymbol{r} \in D_\mathrm{O},$$

where $y(\boldsymbol{r}_i), \boldsymbol{r}_i \in D_\mathrm{M}$ is the scattered field on a sensor located at $\boldsymbol{r}_i$ in the measurement area D_M, $\phi(\boldsymbol{r}), \boldsymbol{r} \in D_\mathrm{O}$ and $\phi_0(\boldsymbol{r}), \boldsymbol{r} \in D_\mathrm{O}$ are the total and the incident field in the object area D_O, and $\mathcal{G}$ is the Green function for the known background medium.

From an algebraic viewpoint, discretization of these equations with a moment method, leads to:

$$\boldsymbol{y} = \boldsymbol{G}_\mathrm{M} \boldsymbol{X} \boldsymbol{\phi}, \tag{1}$$

$$\boldsymbol{\phi} = \boldsymbol{\phi}_0 + \boldsymbol{G}_\mathrm{O} \boldsymbol{X} \boldsymbol{\phi}, \tag{2}$$

where $\boldsymbol{y} \in \mathbb{C}^{n_\mathrm{M}}, \boldsymbol{\phi} \in \mathbb{C}^{n_\mathrm{O}}, \boldsymbol{\phi}_0 \in \mathbb{C}^{n_\mathrm{O}}$, $\boldsymbol{X}$ is a diagonal matrix ($n_\mathrm{O} \times n_\mathrm{O}$) with the components of the vector $\boldsymbol{x} \in \mathbb{C}^{n_\mathrm{O}}$ as diagonal elements, n_O is the number of pixels of the discrete object and n_M is the number of measurement sensors. Note that these notations can be extended for emission from n_S different positions.

Formally, the total field $\boldsymbol{\phi}$ in the object can be expressed from (2) and introduced in (1). It gives an explicit relation between the contrast and the data $\boldsymbol{y} = \mathcal{A}(\boldsymbol{x})$ with:

$$\mathcal{A}(\boldsymbol{x}) = \boldsymbol{G}_\mathrm{M} \boldsymbol{X} \left(\boldsymbol{I} - \boldsymbol{G}_\mathrm{O} \boldsymbol{X}\right)^{-1} \boldsymbol{\phi}_0. \tag{3}$$

The direct problem is modeled equivalently with the coupled equations (1–2) or with the explicit relation (3). Solution of this problem requires the inversion of a $n_\mathrm{O} \times n_\mathrm{O}$ matrix (or solution of a linear system of n_O equation and n_O unknown) and is computationally very expensive. The inverse problem, which we are concerned with consists in determining the contrast $\boldsymbol{x}$ from a given finite set of data $\boldsymbol{y}$, inevitably imperfect with respect to the direct model.

2. BAYESIAN FRAMEWORK

The Bayesian inference is now a common way to handle ill-posed inverse problems in signal and image processing [1]. This general framework can be applied on many ways to the considered problem, according to the choice of different models.

We model the errors on the measurement with an additive zero mean white Gaussian circular noise, which seems to be reasonable in the absence of complementary information.

The *a priori* state of knowledge, that is before any measurement is carried on, is modeled through a probability law $p(\boldsymbol{x}) \propto \exp\{-\mathcal{U}(\boldsymbol{x})\}$ of energy function $\mathcal{U}(\boldsymbol{x})$. The choice of $\mathcal{U}(\boldsymbol{x})$ is a basic point in the Bayesian framework and the Markov Random Fields are classically used for image modeling. The choice of such a function is not the aim of this work and for the sake of simplicity, this energy function is chosen to be convex in the following.

From these assumptions, two formulations can be deduced, depending on whether the contrast $\boldsymbol{x}$ has to be estimated from the data $\boldsymbol{y}$ or both the contrast $\boldsymbol{x}$ and the field in the object $\boldsymbol{\phi}$ have to be estimated – the estimate is defined in the maximum *a posteriori* (MAP) sense.

2.1. First Formulation: Estimation of $\boldsymbol{x}$.

This formulation is straightforward. The solution is defined as the MAP estimate $\boldsymbol{x}_{\text{MAP}} = \arg\max_{\boldsymbol{x}} p(\boldsymbol{x}|\boldsymbol{y})$. From the explicit relation (3) it corresponds to the global minimizer of the criterion

$$\mathcal{J}^{\text{MAP}}(\boldsymbol{x}) = \|\boldsymbol{y} - \mathcal{A}(\boldsymbol{x})\|^2 + \lambda\mathcal{U}(\boldsymbol{x}). \qquad (4)$$

2.2. Second Formulation: Joint estimation of $\boldsymbol{x}$ and $\boldsymbol{\phi}$.

The solution is defined as the joint MAP estimate of $\boldsymbol{x}$ and $\boldsymbol{\phi}$: $(\boldsymbol{x},\boldsymbol{\phi})_{\text{MAP}} = \arg\max_{(\boldsymbol{x},\boldsymbol{\phi})} p(\boldsymbol{x},\boldsymbol{\phi}|\boldsymbol{y})$, where

$$p(\boldsymbol{x},\boldsymbol{\phi}|\boldsymbol{y}) = \frac{p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{\phi})p(\boldsymbol{\phi}|\boldsymbol{x})p(\boldsymbol{x})}{p(\boldsymbol{y})}. \qquad (5)$$

In this relation, $p(\boldsymbol{y})$ is a constant and $p(\boldsymbol{x})$ has already been defined so only the first two numerator terms are to be specified:

• Using (1), with the considered error model, first term can be written:

$$p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{\phi}) \propto \exp\left\{-\frac{1}{\sigma_b^2}\|\boldsymbol{y} - \boldsymbol{G}_{\text{M}}\boldsymbol{X}\boldsymbol{\phi}\|^2\right\};$$

• Second term corresponds to the probability law of $\boldsymbol{\phi}$ for a known $\boldsymbol{x}$. As $\boldsymbol{\phi}$ is the total field in the object, it is uniquely determined for a given $\boldsymbol{x}$ by (2). Thus, if δ denotes the Dirac distribution:

$$p(\boldsymbol{\phi}|\boldsymbol{x}) = \delta(\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi});$$

Finally, using these expressions, the posterior probability law can be written:

$$p(\boldsymbol{x},\boldsymbol{\phi}|\boldsymbol{y}) \propto \exp\left\{-\frac{1}{\sigma_b^2}\|\boldsymbol{y} - \boldsymbol{G}_{\text{M}}\boldsymbol{X}\boldsymbol{\phi}\|^2 - \mu\mathcal{U}(\boldsymbol{x})\right\} \delta(\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi}).$$

So the MAP estimate of $(\boldsymbol{x},\boldsymbol{\phi})$ is the minimizer of the criterion:

$$\mathcal{J}_c^{\text{MAP}}(\boldsymbol{x},\boldsymbol{\phi}) = \|\boldsymbol{y} - \boldsymbol{G}_{\text{M}}\boldsymbol{X}\boldsymbol{\phi}\|^2 + \lambda\mathcal{U}(\boldsymbol{x}), \qquad (6)$$

subject to the constraint:

$$\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi} = 0. \qquad (7)$$

2.3. A Computational Challenge

The regularized solution has been defined as the solution of a constrained or an unconstrained optimization problem. These two distinct formulations are equivalent in the sense that they define the same solution (for $\boldsymbol{x}$), but one may think about the use of different techniques to solve them.

The Bayesian framework is actually not necessary to define the solution as the minimizer of (4). Indeed, this criterion is simply a penalized least square criterion which can be taken into account in a deterministic framework. On the other hand, the definition of the joint solution as the minimum of (6) under constraint (7) is not straightforward to set from deterministic arguments and other joint criteria are often proposed, which will be studied in § 4.

Due to the non-linearity of the direct problem, it is easy to show that criteria (4) and (6) are not convex functions. Thus, even if the prior information is modeled with a convex energy function, these criteria may have local minima. From simulation experiments, appearance of local minima is closely linked to a high contrast value, a limited number of measurements and a low signal-to-noise ratio. Thus computation of the solution may be a cumbersome task, especially in these *difficult* configurations. However, the problem seems to be less difficult in more favorable configurations.

In the simplest case, a linear approximation (*e.g.*, Born) of the direct model can be considered, which seems to be sufficient for some few applications [2]. However, the linear case has already been extensively studied and this paper is concerned with nonlinear DT. In the following sections, we propose a classification of existing methods for solving this problem.

3. SUCCESSIVE LINEARIZATIONS

Methods of the *first class* consider iteratively linear approximations of the direct model, which leads to successively resolve linear inverse problems. Different methods of this class have been proposed in the literature to solve the nonlinear DT problem [4, 5, 6]. In the proposed Bayesian framework, one may find strong correspondences between them, as shown in [7]. It is also emphasized that none of these methods were efficient both in terms of linear approximation and of regularization. In fact, regularization has been introduced to stabilize the solution of each linear problem, but not to regularize the nonlinear problem as a whole.

Thus we have proposed a successive linearization algorithm specifically designed to minimize the MAP criterion (4). At each iteration n, with corresponding solution $\boldsymbol{x}_n$, the first order Taylor series expansion of $\mathcal{A}$ near $\boldsymbol{x}_n$ is taken into account (strictly speaking, one has to account for the Taylor series expansion of the real and imaginary parts of $\mathcal{A}$ to define such a relation). Thus minimization of $\mathcal{J}^{\text{MAP}}$ can be performed with successive linearizations of $\mathcal{A}$:

Initialize $n = 0, \boldsymbol{x}_0$.

Iterate for $n = 1, 2\ldots$ until convergence towards a stationary point:

1 Compute the matrix $\boldsymbol{A}_n$ corresponding to the linear approximation of $\mathcal{A}$ near current solution $\boldsymbol{x}_n$;

2 Compute $\boldsymbol{x}_{n+1} = \arg\min_{\boldsymbol{x}} \mathcal{J}_n(\boldsymbol{x})$ with $\mathcal{J}_n(\boldsymbol{x}) = \|\boldsymbol{y} - \mathcal{A}(\boldsymbol{x}_n) - \boldsymbol{A}_n(\boldsymbol{x} - \boldsymbol{x}_n)\|^2 + \lambda\mathcal{U}(\boldsymbol{x})$.

Note that for convex energy functions $\mathcal{U}$, all criteria to be minimized are convex functions and consequently have a unique global minimum which can be computed using a gradient descent technique.

At each step, $\mathcal{J}^{\text{MAP}}$ is approximated by a convex criterion $\mathcal{J}_n$ with same value at $\boldsymbol{x}_n$ and same slope at this point. Of course, there is no convergence guarantee and the algorithm could diverge. However, if it converges towards $\boldsymbol{x}_\infty$, this point corresponds to a

stationary point of the criterion $\mathcal{J}^{\text{MAP}}$ (*i.e.,* such that $\nabla_{\boldsymbol{x}}\mathcal{J}^{\text{MAP}}(\boldsymbol{x}_\infty) = \boldsymbol{0}$). But the possible convergence and the reached stationary point are dependent upon the initialization of the algorithm.

The computational load of such an algorithm is moderate as it does not require the computation of the direct problem, but only of a linear approximate, during the minimization step. However, the computation of the approximating matrix $\boldsymbol{A}_n$ at each iteration, requires solution of the direct problem. Such an algorithm can be used to compute the MAP solution with a relative low computational cost, when the criterion, even not convex, seems not to have any local minima.

4. MINIMIZATION OF A JOINT CRITERION

Some recently proposed methods [8, 9, 10] – methods of the *second class* – define the solution as the minimizer of a criterion jointly on the contrast $\boldsymbol{x}$ and the field in the object $\boldsymbol{\phi}$, with the following generic form:

$$\begin{aligned} \boldsymbol{F}(\boldsymbol{x},\boldsymbol{\phi}) &= \|\boldsymbol{y} - \boldsymbol{G}_{\text{M}}\boldsymbol{X}\boldsymbol{\phi}\|^2 \qquad (8)\\ &+ \alpha\,\|\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi}\|^2 + \lambda\mathcal{U}(\boldsymbol{x},\boldsymbol{\phi}). \end{aligned}$$

Such a definition is very easy to understand intuitively: it corresponds to jointly minimizing the errors on (1) and (2) and, as the problem is ill-posed, a penalization term on the unknowns is added to regularize it.

The proposed methods differ on several points:

- Different value has been proposed for parameter α.
- Differences appear on the regularization term. First, no regularization was introduced [8, 9]. Then, it has been proposed to regularize both on $\boldsymbol{x}$ and $\boldsymbol{\phi}$, with an energy function $\mathcal{U}(\boldsymbol{x},\boldsymbol{\phi})$ [10, 11]. Finally, a single regularization term on $\boldsymbol{x}$ was accounted for [12, 13].
- The methods also differ in the techniques used to compute the solution. Usual gradient type local minimization techniques have been used [9, 11] as well as local techniques specially designed for such a criterion [8] and global minimization techniques such as Simulated Annealing [12].

Note that such a method never requires solution of the direct problem – which was one of their main objectives – so it is of relative low computation cost. On the other hand, the number of unknown is multiplied by $n_{\text{S}} + 1$, as the object $\boldsymbol{x}$ and the field in the object for each incident wave have to be determinate.

4.1. Bayesian viewpoint

Recall from § 2.2 that joint estimation of $\boldsymbol{x}$ and $\boldsymbol{\phi}$ leads to minimization of (6) subject to constraint (7). In this framework, criterion (8) can be understood as the Lagrangian of this constrained optimization problem (for a scalar constraint: $\|\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi}\|^2 = 0$); or has a penalization of (6) with this constraint. In both cases, the choice of parameter α (the Lagrange parameter) is important: on one hand, the value of α should be high enough to enforce the constraint; on the other hand, the criterion may become numerically insensitive to the data if α is too high. Note that α has been fixed intuitively in the different methods.

Moreover, this viewpoint gives indications for regularizing such a criterion with an energy function $\mathcal{U}(\boldsymbol{x})$. Using Bayes rule for the considered model of errors on measurements, we can see on (5) that there is no need to introduce prior model on $\boldsymbol{\phi}$.

Another Bayesian interpretation of this criterion has been given in [12]. It is proposed to account for additive Gaussian models for errors on both coupled equations (1–2), so that the joint MAP estimate of $\boldsymbol{x}$ and $\boldsymbol{\phi}$ minimizes a criterion of form (8). However, it can be shown [14] that to obtain such a criterion, the error on the measurement should be taken correlated to the unknown $\boldsymbol{x}$ or $\boldsymbol{\phi}$, which seems to be a strong and unjustified hypothesis.

Of course, the minimization of (8) is not equivalent to compute the MAP estimate has defined in § 2.2. However, it is easy to show that if $(\boldsymbol{x},\boldsymbol{\phi})$ is a local minimum of (8) and that constraint (7) is verified, then $\boldsymbol{x}$ corresponds to a local extremum of the unconstrained criterion $\mathcal{J}^{\text{MAP}}$.

4.2. Proposed algorithm

We proposed a new algorithm to deal with this constrained optimization problem [14]. This algorithm is a strict application of the method of multipliers [3] – which uses the augmented Lagrangian – and can be summarized as follows:

Initialize $n = 0, \boldsymbol{\mu}_0 = \boldsymbol{0}, \alpha_0$

Iterate for $n = 1, 2\ldots$ until convergence towards a stationary point:

1 Compute $(\boldsymbol{x}_n,\boldsymbol{\phi}_n)$ minimizing the augmented Lagrangian: $\mathcal{L}_{\alpha_n}((\boldsymbol{x},\boldsymbol{\phi}),\boldsymbol{\mu}_n) =$
$$\|\boldsymbol{y} - \boldsymbol{G}_{\text{M}}\boldsymbol{X}\boldsymbol{\phi}\|^2 + \boldsymbol{\mu}_n^{\text{t}}(\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi})$$
$$+ \quad \alpha_n\|\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi}\|^2 + \lambda\mathcal{U}(\boldsymbol{x});$$

2 Update parameters: $\alpha_{n+1} = \beta\alpha_n$ (β fixed) and $\boldsymbol{\mu}_{n+1} = \boldsymbol{\mu}_n + \alpha_n(\boldsymbol{\phi} - \boldsymbol{\phi}_0 - \boldsymbol{G}_{\text{O}}\boldsymbol{X}\boldsymbol{\phi})$.

We have used a gradient descent technique to perform the minimization of the augmented Lagrangian, so the reached minimum at each step is only a local minimum. Such an algorithm, when converges, guarantees to reach a local minimum of (6) which verifies the constraint (7), but the convergence is not guaranteed. However, the first step of this algorithm corresponds to the minimization of (8), and the use of additional iterations has been shown to decrease the error on the constraint and to improve the solution.

5. DIRECT MINIMIZATION OF $\mathcal{J}^{\text{MAP}}$

Different methods – *Third class* methods – minimize the least square criterion, possibly taking into account a regularization term (*e.g.* [15, 16, 17]); which correspond to directly minimize the MAP criterion (4). The computation of this criterion is very expensive as it requires solution of the direct problem. Thus, two cases should be distinguished:

- In *easy configurations*, when (4) – even nonconvex – seems to have a unique minimum, a local optimization technique could be used [16, 17]. However, we have shown that such a minimization could be done with less computationally expensive *first* and *second classes* methods; so one should avoid minimizing directly (4).
- In *difficult configurations*, when such local minima exist, *first* and *second classes* methods discussed § 4 and § 3 fail. In this case, a global optimization technique could be used to minimize (4).

Simulated Annealing has been used in [15], but such a technique is practically inextricable because of the high computation cost of the criterion and the

large support of the operator $\mathcal{A}$. Instead, we proposed two less expensive deterministic algorithms to try to reach the global minimum:

• The first algorithm [18], is based on a Graduated Non Convexity scheme (GNC). It consists in approximating globally the criterion to be minimized with a sequence of criteria, which converges towards it, by taking care to choose the first one to be convex. Then, each criterion is minimized locally, using as starting point the minimum of the previous criterion.

• The second one is an Iterated Conditional Mode (ICM) algorithm which performs the minimization by iteratively updating each pixel of the object. Such an algorithm can be efficiently implemented [19] using special properties of the criterion (4).

None of these algorithms are guaranteed to converge towards the global minimum but both have given satisfactory results in configurations where a gradient type algorithm gets stuck in a local minimum.

6. CONCLUSION

The study of DT in the Bayesian estimation framework allowed us to define a regularized solution to this nonlinear inverse problem.

We have proposed a classification of most of the existing methods in terms of algorithms to compute this solution. Three classes have been distinguished.

Methods of the *first class* correspond to successive approximations of the nonlinear object/data relation with a linear one. At each iteration, a minimization step can be performed at a relatively low computation cost, but an update step requires solution of the computationally expensive direct problem.

Methods of the *second class* define the solution as the joint minimizer of a criterion depending both on the object and on the total field in the object. These methods do not require solution of the direct problem and the criterion to be minimized is of equivalent computation order to the *first class* methods criteria. However, the number of unknowns is greater, especially if the measured scattered field corresponds to different incident waves.

Third class methods directly minimize the MAP criterion depending on the object. As such methods are much more expensive than the others, they should be used when the others fail, in particular in presence of local minima in the criterion. In this case, global optimization methods of this *third class* can give satisfactory solutions while methods of the *first* and *second classes* get stuck in local minima.

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