# SELECTIVE WAVELET COEFFICIENT SOFT-THRESHOLDING SCHEME FOR SPECKLE NOISE REDUCTION IN SAR IMAGES

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## ABSTRACT

Speckle noise is one characteristic of Synthetic Aperture Radar (SAR) images. The primary goal of speckle smoothing of SAR images is to reduce the speckle noise without sacrificing information content. Various speckle filters have been devised to smooth speckle in the spatial domain. In this paper, we perform speckle reduction in the wavelet domain. A hierarchical correlation is defined which takes into account both the inter- and intra-band correlation among wavelet coefficients. According to this definition, the correlation values at edge positions are larger than those for non-edge positions. We use this correlation map to distinguish edge coefficients from noise coefficients and thus perform selective soft-thresholding on the wavelet coefficients. The proposed method is applied to airborne SAR images and the results are compared with Donoho's original soft-thresholding and the well-known Lee multiplicative speckle filter. Test results show that this method can substantially smooth noise while preserving major edge structures in images.

## 1. INTRODUCTION

Synthetic Aperture Radars (SAR) are active imaging systems widely used in remote sensing applications. SAR systems are characterized by their high image resolution and all-weather operating ability, but SAR images also suffer from the notorious speckle noise, a chaotic phenomenon that results from coherent imaging [1]. Speckle noise can obscure scene content and strongly reduce the ability for object recognition. Thus speckle noise reduction has long been a central problem in SAR image processing.

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Various speckle smoothing algorithms have been proposed which adapt the parameters of filters to the local noise statistics in the spatial domain. The Lee multiplicative speckle filter is perhaps the most famous among these[2]. This filter can effectively reduce speckle in homogeneous areas, but does not perform as well in the edge regions. The selection of the processing window size in the Lee filter is a compromise between noise smoothing and edge preserving; a large window size results in less prominent noise in edge areas, but the edges are somewhat blurred.

In this paper, we present a new speckle noise reduction algorithm performed in the wavelet transform domain. Wavelet domain soft-thresholding, originally proposed by Donoho[5], has been found to be an effective method for noise reduction. In the original algorithm (called "Donoho's overall soft-thresholding" in this paper), all the wavelet coefficients are soft-threshold processed. While speckle noise is reduced by this method, the subtle edges are also destroyed by this indiscriminate processing. The algorithm proposed in this paper attempts to overcome this problem by performing selective wavelet coefficient soft-thresholding, thus better preserving edge structures.

## 2. REVIEW OF THE WAVELET TRANSFORM AND SOFT-THRESHOLDING

The essential concept of the wavelet transform of a function is to represent the function as a superposition of wavelet basis functions. The wavelet basis functions are generated from a single mother wavelet function $\psi(t)$ by dilations and translations. A discrete orthogonal wavelet basis would consist of the following functions:

$$\psi_{m,n}(t) = 2^{m/2} \cdot \psi(2^{-m}t - n) \tag{1}$$

where $m$ and $n$ are indexing integers, and $t$ is a variable denoting time or space. In addition to $\psi(t)$, there is usually another function $\phi(t)$ called the scaling function which complements $\psi(t)$ in representing a signal at the same scale. The discrete wavelet transform can be implemented by filter banks. Suppose $h(n)$ and $g(n)$ are filters associated with a discrete orthogonal wavelet basis $\phi(t)$ and $\psi(t)$, respectively. A sampled signal $s_0(n)$ can be decomposed as:

$$s_1(n) = \sum_k h(2n-k)s_0(k), \quad c_1(n) = \sum_k g(2n-k)s_0(k) \tag{2}$$

This operation can be repeated on the signal $s_1(n)$ and so on up to scale $j$. In this case the signal set $c_1 \cup c_2 \cup c_3 \cup \ldots \cup c_i \cup c_j \cup s_j$ provides a lossless representation of $s_0(n)$. A more comprehensive discussion of the wavelet transform may be found in [3] or [4].

The attraction of wavelet transform in image processing lies with two facts. First, a wavelet transform decomposes an image into a multiresolution representation. Each subimage can be handled with different processing parameters. Second, a wavelet transform possess good spatial-frequency resolution, thus it can track rapid changes in image content effectively.

Noise filtering is one application of wavelet domain image processing. Donoho [5] proposed a wavelet shrinkage procedure for the optimum recovery of a signal from a noisy data set. Let $y_i = x_i + n_i, i = 0, \cdots, N-1$, where $x_i$ is the *true* signal, and $n_i$ is white Gaussian noise. Let $\hat{x}$ be the estimate of $x$. The goal is to optimize the mean-square error

$$\frac{1}{N}E||\hat{x} - x||_2^2 = \frac{1}{N}\sum_{i=0}^{N-1} E(\hat{x}_i - x_i)^2 \tag{3}$$

subject to the side condition that with high probability, $\hat{x}$ is at least as smooth as $x$. A wavelet transform is applied to the data set and the noise standard deviation $\sigma$ of the coefficients is estimated. A universal threshold can be calculated by

$$t_n = \sqrt{2\log(N)}\,\sigma/\sqrt{N} \tag{4}$$

and soft-thresholding is applied to each wavelet coefficient $c_m(n)$ to obtain the new coefficient $\hat{c}_m(n)$ by:

$$\hat{c}_m(n) = syn(c_m(n))(|c_m(n)| - t_n)_+ \tag{5}$$

where $(x)_+$ takes the value $x$ for positive $x$ and zero otherwise. The de-noised signal $\hat{x}_i$ can then be obtained by the inverse wavelet transform.

Applications of the soft-thresholding method for noise reduction has been reported by [6, 7] with promising results. However, the resulting images usually have a blurred and dimmed appearance because the edge pixels as well as noisy pixels are equally smoothed. In order to retain both the large structures and the subtle details, we have refined Donoho's overall soft-thresholding approach.

## 3. THE SELECTIVE WAVELET COEFFICIENT SOFT-THRESHOLDING METHOD

To overcome the problem of Donoho's overall soft-thresholding method, we need to identify edge pixels in the wavelet domain and protect them from soft-thresholding. We know wavelet coefficients with larger values usually indicate the positions of rapid changes (i.e., edges) in an image, and small coefficients usually correspond to detail information. While this is always the case for clean images, wavelet coefficients for noisy images are inevitably contaminated by noise and can hardly be used to identify edges directly. Thus, alternative methods should be investigated.

In this work, we adopt the common multiplicative model of speckle noise. Let $y(i,j) = x(i,j)n(i,j)$, where $y(i,j)$ is the $(i,j)$th intensity or amplitude of a SAR image pixel, $x(i,j)$ is the noise-free quantity at $(i,j)$ and $n(i,j)$ is the speckle noise characterized by a distribution with a unit mean ($E[n] = 1$) and a standard deviation $\sigma_v$. We define

$$\sigma_v = \frac{\sqrt{var(y)}}{E[y]} \tag{6}$$

$\sigma_v$ can be used as a measure of speckle strength. The *multiplicative* nature of the speckle noise has been verified by scatter plots of sample standard deviation versus sample mean produced in many homogeneous areas of SAR images [2]. For example, in featureless regions of a four-look SAR image, $\sigma_v$ is approximately 0.26.

A logarithmic transform is employed to convert the multiplicative speckle noise into additive noise. Since the wavelet transform is a linear operation, it will not change the log-transformed noise statistics. Thus, we can expect the noise in the logarithmic preprocessed images to be manifest as additive noise.

Although the orthogonal wavelet transform does a good job in decorrelating an image, the resulting wavelet coefficients are not totally uncorrelated. As proved by Dijkerman and Mazumdar, the correlation between orthogonal wavelet coefficients decreases exponentially quickly across scales and hyperbolically along time (space) [8]. Therefore, we can make use of this short term correlation to select informative image features from the noise which, after the orthogonal transform remains uncorrelated.

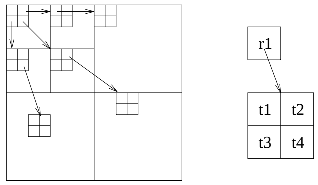



Figure 1: Illustration of wavelet quadtrees of an 3-level wavelet decomposition.

We note that while large structures can be found in many scales, small image details can only be revealed in several fine scales. Thus we calculate inter-band correlation involving only two adjacent scales. Besides, when an orthogonal wavelet transform is adopted, horizontal, vertical and diagonal subimages will be generated which are quite different in their edge extraction abilities, i.e., oriented edges can only be visible in certain subimages. This suggest that subimages of different orientations should be treated separately. Therefore, in the following context, the same procedure will be applied to the horizontal (H), vertical(V) and diagonal (D) subimages separately.

From the above discussions, we can introduce the concept of hierarchical correlation which takes into account both the near neighbor intra-band correlation and the adjacent inter-band correlation along partial *quadtrees*(Fig.1) which is a group of five wavelet coefficients corresponding to the same spatial location. For any two levels in the wavelet decomposition, the hierarchical correlation is defined as

$$correlation = \sqrt{r_1 \cdot intra} \tag{7}$$

$$intra = \begin{cases} max(\sqrt{t_1 \cdot t_2}, \sqrt{t_3 \cdot t_4}) & H - orient \\ max(\sqrt{t_1 \cdot t_3}, \sqrt{t_2 \cdot t_4}) & V - orient \\ max(\sqrt{t_1 \cdot t_4}, \sqrt{t_2 \cdot t_3}) & D - orient \end{cases} \tag{8}$$

where $r_1$ is one coefficient in the coarser scale and $t_1, t_2, t_3$ and $t_4$ are four coefficients in the next finer scale. Notice that the intra-band correlation is computed differently according to the orientation of the subimages involved. Oriented edges will generate larger correlation values and will be detected by this definition. A *correlation map* whose size is a quarter of the original image is then obtained because coefficients in the finest scale have no descendants. Large coefficients in this map indicate the position of edges in the original image, and zero coefficients correspond to smooth areas. The left-top part of the correlation map which corresponds to the lowpass subimage in the wavelet decomposition will not be computed since we consider the coarsest resolution subimages are *clean* enough and as such, could be left unprocessed.

Our correlation definition allows fine structures which do not appear as local maxima to be revealed in the correlation map. This correlation map can then be used as an edge position indicator in the wavelet domain. Quadtrees which are not selected as edges for certain correlation thresholds will be smoothed as noise. The algorithm is designed as an iterative one starting with a relatively small correlation threshold which is increased on each iteration to gradually eliminate noise (Fig. 2). The complete processing procedure is given as follows:

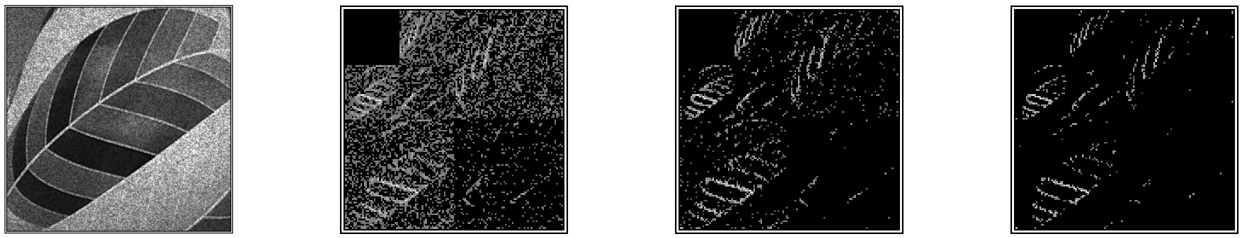
1. Perform logarithmic operation on the original image to convert multiplicative noise into additive noise.
2. Apply orthogonal wavelet transform with maximum possible decomposition level.
3. (a) Compute correlation map.
   (b) Perform selective soft-thresholding on non-edge quadtrees.
   (c) Stop if a stopping conditions is reached, else increase the threshold for edge detection and return to 3a.
4. Apply inverse wavelet transform.
5. Perform exponential operation.

Two thresholds are employed in the above algorithm. One is that used to determine edges from the correlation map. Another threshold is used for the soft-thresholding. From our experience, the universal threshold obtained from Donoho's formula tends to oversmooth images. Currently there is no other well defined criterion for threshold selection. In this algorithm, we use the variance of the finest scale diagonal oriented subimage as this threshold, and decrease it by a small value (e.g., 0.5) when it is used to process larger scale subimages.

There are two ways to stop this algorithm. One is to specify the number of iteration times. We have found that 10 − 15 iterations with a threshold increment of 0.5 can yield good noise reduction results for most of the test images used in our research. Alternatively, the algorithm can be stopped when a certain percentage of the image pixels within an image are determined as edges. Thus this algorithm trades off preserving image details and reducing noise. These two conditions should be adjusted for different images according to image type and complexity.

## 4. TEST RESULTS AND DISCUSSIONS

Fig. 3 shows an original airborne SAR amplitude image(Fig. 3(a)) and three filtered images (Fig. 3(b)-(d)). Fig. 3(b) is produced by Donoho's overall soft-thresholding with threshold=3. Fig. 3(c) shows the result of the proposed selective soft-thresholding method



(a) (b) (c) (d)

Figure 2: Correlation maps of a 4-level decomposed noisy image (a) after 2 iterations (b), 3 iterations (c) or 4 iterations(d). Major edges are gradually made visible along with the elimination of noise.

after 10 iterations and with a starting correlation threshold=1. We can see that the soft-thresholding technique can effectively reduce the speckle noise. However, Donoho's overall approach tends to smooth both edge and noise pixels. The filtering result obtained from selective soft-thresholding has a much brighter appearance with enhanced edge sharpness, and with the thin edges well preserved. The smoothed image produced by the Lee multiplicative speckle filter is provided for comparison(Fig. 3(d)). Since this test image contains many fine structures, a small mask size (3×3) is chosen to prevent over-smoothing. While this filter works fairly well in homogeneous areas, substantial noise is left in the edge regions.

Because there are no noise -free versions of the SAR images, we cannot use PSNR or other similar objective measures to evaluate the noise smoothing performance. Instead, we manually choose several small regions (11 × 11) from the images which appear to be homogeneous to evaluate the noise smoothing effect in these regions. The mean, variance and $\sigma_v$ measures for two of these regions are shown in Table 1. In addition to these homogeneous regions, we also select two regions which contain edge activity and the measures are also shown in Table 1. For each smoothed image, we expect small variance and $\sigma_v$ values in the flat regions and large variance and $\sigma_v$ values in the edge regions. Again, we include those measures for the images obtained from the Lee filter for comparison. We see that for this and other test images not shown here, the proposed filter performs better overall than the Lee filter in both noise smoothing and edge preservation.

We should note that although the proposed filter produces good results on the test images, it has some limitations. First, it is capable of detecting and enhancing line features in the images, but it does not perform equally well on features with short duration such as point targets. Second, currently the proposed method stops after having finished a certain number of iterations. For images with many edge activities, we perform fewer iterations to preserve image details while for relatively simple images, we can do more iterations. For images with spatially varying complexity, we have to trade off the noise smoothing and detail preservation by using a moderate times of iterations. The issue of adapting iteration times to image statistics deserves further study.

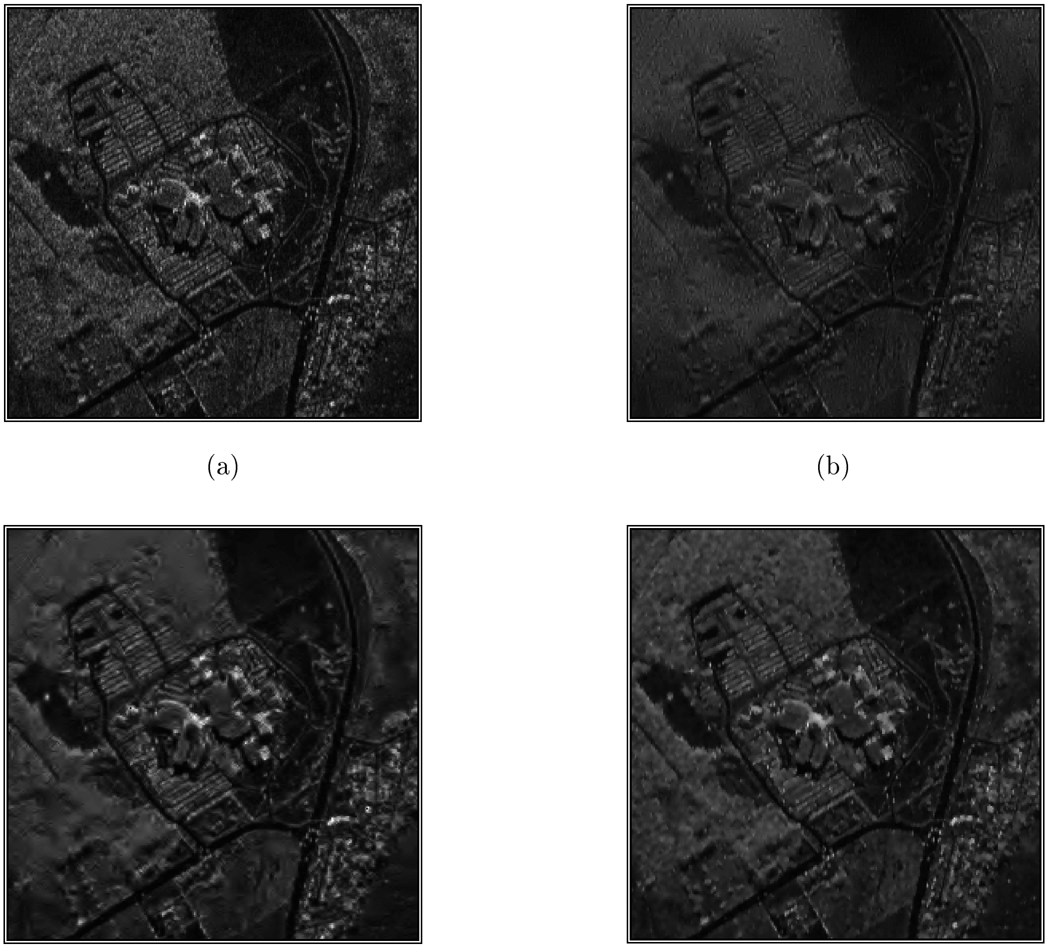
| image | area1(flat) | | | area2(flat) | | |
|---|---|---|---|---|---|---|
| | mean | var | $\sigma_v$ | mean | var | $\sigma_v$ |
| speckle | 14.45 | 4.28 | 0.296 | 58.72 | 14.48 | 0.247 |
| Lee | 13.88 | 1.61 | 0.116 | 57.76 | 4.12 | 0.071 |
| overall | 12.08 | 1.62 | 0.134 | 52.80 | 5.43 | 0.103 |
| selective | 12.93 | 1.09 | 0.084 | 53.87 | 3.05 | 0.057 |

| image | area3(edge) | | | area4(edge) | | |
|---|---|---|---|---|---|---|
| | mean | var | $\sigma_v$ | mean | var | $\sigma_v$ |
| speckle | 50.95 | 20.01 | 0.393 | 22.06 | 13.29 | 0.602 |
| Lee | 50.39 | 14.32 | 0.284 | 21.37 | 10.90 | 0.510 |
| overall | 46.69 | 8.94 | 0.191 | 19.58 | 8.72 | 0.445 |
| selective | 48.54 | 16.25 | 0.335 | 20.89 | 11.64 | 0.557 |

Table 1: Quantitative measures for noise smoothing and edge preserving of *industry* image.

## 5. CONCLUSIONS

In this paper, we have proposed a new wavelet domain speckle noise reduction method. The edge pixels are identified from the hierarchical correlation map by exploiting both the inter- and intra-band correlation among the wavelet coefficients; thus edge pixels are protected from soft-thresholding. Compared with Donoho's overall soft-thresholding noise reduction approach and the Lee multiplicative filter, the proposed method can achieve better smoothing of the speckle noise and preservation of the subtle but distinguishable features.



(a) (b) (c) (d)

Figure 3: Comparison of speckle reduction results: (a) Original SAR image (4 - look); (b) Speckle reduction using Donoho's overall soft-thresholding, threshold=3; (c) Speckle reduction using selective soft-thresholding, threshold=1, 10 iterations; (d) Speckle reduction using Lee Multiplicative filter, mask size=$3 \times 3$.

## 6. REFERENCES


[1] J.W. Goodman, "Some Fundamental Properties of Speckle", *J. Opt. Soc. Am*, vol. 66, pp. 1145–1150, 1976.

[2] J.S. Lee, "Digital Image Enhancement and Noise Filtering by Use of Local Statistics", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol. 2, pp. 165–168, 1980.

[3] I. Daudbechies, *"Ten Lectures on Wavelets"*, CBMS-NSF Regional Conference Series, SIAM, 1992.

[4] S. Mallat, "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation", *IEEE Trans. on Pattern Analysis and Machine Intelligence*, vol.11, pp. 674–693, 1989.

[5] D.L. Donoho, "De-noising by Soft-thresholding", *IEEE Trans. on Information Theory*, vol. 41, pp. 613–626, 1995.

[6] J.E. Odegard, *et al*, " Speckle Reduction via Wavelet Shrinkage with Application to SAR based ATD/R", *SPIE Math. Imaging: Wavelet Applications in Signal and Image Processing.*, vol. 2303, pp. 333–344, 1994.

[7] Y. Li, C. Moloney, "SAR Image Compression Using Wavelet Transform and Soft-thresholding", *Proceedings of Canadian Conference on Electrical and Computer Engineering*, vol.1, pp. 387–390, 1997.

[8] R.W. Dijkerman, R.R. Mazumder, "On the Correlation Structure of the Wavelet Coefficients of Fractional Brownian Motion", *IEEE Trans. on Information Theory*, vol. 40, pp. 1609–1612, 1994.