Fuzzy Weighted Median Filters

Akira Taguchi and *Susumu Takaku*

Dept.of Electrical and Electronic Engineering, Musashi Institute of Technology
1-28-1,Tamazutsumi,Setagaya-ku,Tokyo 158, Japan
Tel:+81-3-3703-3111 Fax:+81-3-5707-2174
E-mail:taguchi@ipc.musashi-tech.ac.jp

ABSTRACT

Stack filters are a class on nonlinear filters and include rank order filters, morphological filters, weighted median filters, and so on. The stack filter is defined by a Boolean function. The output of Boolean function is restricted two values(i.e., 0 or 1). We attempt to enlarge class of stack filter by defining the output of Boolean function from 0 to 1 continuously. We call this filter the fuzzy stack filter.

We have already proposed fuzzy weighted median (FWM) filters which are important class of fuzzy stack filters, and shown a simple design method of those. However, this design method imposed restrictions on the class of FWM filter which is able to be design.

In this paper, we propose a novel design method of FWM filters by using LMS algorithm. We can design the optimal FWM filters form all class of those by the proposed method.

1 Introduction

Stack filters [4] were proposed as a generalization of rank order filters in an effort to increase the variety of available nonlinear operations. A stack filter is a sliding window nonlinear filter whose output at each window position is the result of a superposition of a stack of positive Boolean function operating on threshold version of the samples in the filter's window. The output of Boolean function is restricted two values ("0" or "1"). Intuitively, one would expect better performance for stack filters, if the output of Boolean function is defined form 0 to 1 continuously. This Boolean function are called fuzzy Boolean function [1],[2].

The weighted median (WM) filters, whose weights each signal sample within the filter window by some nonnegative weight, is an extension of the median filter[1].It was shown that WM smoothers are quite robust for different type of noise and, at the same time, are able to preserve edges. Several research works have been reported to optimize WM filters under the mean absolute error or the mean square error (MSE) criterion. WM filters are nonlinear filters belonging to the class of stack filters[4].

In [6], we have proposed the fuzzy WM(FWM) filter which is defined by fuzzy Boolean function. FWM filters include WM filters and weighted average(i.e. linear FIR) filters. It is well know that WM filters can remove impulsive noise effectively. However, the non-impulsive noise removing ability of WM filters are inferior to that of weighted average (WA) filters. The proposed FWM filters can be designed as a moderate stand between WM filters and WA filters. Thus, the mixed noise removing ability of FWM filters is superior to that of WM filters and WA filters. The FWM filter is synthesized by two factors, those are fuzzy Boolean function and filter weights. We approximated the fuzzy Boolean function by sigmoidal function Furthermore, we approximated weights of the FWM filter by the S-type function which is used in the field of fuzzy theory. As a result, the FWM filter is defined by only four parameters. The proposed filter can be derived by LMS algorithm under the mean squared error criterion. We call the FWM filter which is designed by this method, the simple type of FWM (SFWM) filter.

In this paper, we propose a novel design method of FWM filters. Fuzzy Boolean function of the proposed FWM filter defined by the modified sigmoidal function with three parameters , all weights of the FWM filter are designed independently. The proposed FWM filter is also easily designed by using LMS algorithm. And we call the FWM filter which is designed by the proposed method, the general type of FWM (GFWM) filter. Experimental result in image restoration demonstrate that GFWM filters have properties of not only WM filters but also weighted average filters. Furthermore, GFWM filters are superior to WM filters, WA filters and SFWM filters, especially, in mixed noise environment.

2 WM FILTERS EXPRESSED BY STACK FILTERS

The output of WM filters with 2N+1 window is given by

$$y(i) = [W_{-N} \Diamond \mathrm{x(i-N)}, \ldots, \mathrm{W}_0 \Diamond \mathrm{x(i)}, \ldots, \mathrm{W}_N \Diamond \mathrm{x(i+N)}] \quad (1)$$

Where $x(i)$ is M-valued input signal and the symbol $\Diamond$ is used to denote duplication. The weights of WM filters are assumed real positive value [3].

WM filters are include in stack filters. Let $r^{(m)}(i)$ is defined as

$$r^{(m)}(i) = \sum_{j=-N}^{N} W_j \cdot x^{(m)}(i+j) \Big/ \sum_{j=-N}^{N} W_j \tag{2}$$

where $x^{(m)}(i)$ is given by

$$x^{(m)}(i) = \begin{cases} 1: & \text{if } x(i) \geq m \\ 0: & \text{otherwise} \end{cases}$$

$$(m = 1, \ldots, M-1) \tag{3}$$

The Boolean function of WM filters is shown as

$$f_{WM}(r^{(m)}(i)) = \begin{cases} 1: & \text{if } r^{(m)}(i) > 0.5 \\ 0: & \text{otherwise} \end{cases} \tag{4}$$

The output of the WM filter is given by

$$y(i) = \sum_{m=1}^{M-1} f_{WM}(r^{(m)}(i)) \tag{5}$$

3 FWM FILTERS

3.1 SFWM filters

FWM filters are defined by fuzzy Boolean function $f_{FWM}(\cdot)$. If $f_{FWM}(r^{(m)}(i))$ is monotonous increasing for $r^{(m)}(i)$, $f_{FWM}(\cdot)$ processes the stacking property.

In [6], the fuzzy Boolean function was approximated by the sigmaidal function. Thus, fuzzy Boolean function of SFWM filters is given by

$$f_{SFWM}(r^{(m)}(i)) = \begin{cases} 1/[1+\exp\{-\alpha_1(r^{(m)}(i)-\beta)\}] & \text{if } r^{(m)}(i) > \beta \\ 1/[1+\exp\{-\alpha_2(r^{(m)}(i)-\beta)\}] & \text{otherwise} \end{cases} \tag{6}$$

The fuzzy Boolean function is defined by only three parameters. The weights of SFWM filters are defined by

$$W_j = \begin{cases} 1 - 2j^2/\gamma^2 & \text{if } 0 \leq |j| \leq \gamma/2 \\ 2(|j|-\gamma)^2/\gamma^2 & \text{if } \gamma/2 < |j| < \gamma \\ 0 & \text{if } \gamma \leq |j| \end{cases} \tag{7}$$

where $|j|$ is the distance from the center point. This function is S-type function which is popular in the field of fuzzy theory. All weights of the SFWM filter can be decided only one parameter and are easily to be derived.

3.2 GFWM filters

In this paper, we attempt to approximate the fuzzy Boolean function $f_{FWM}(\cdot)$ by the modified sigmoidal function on the condition that $f_{FWM}(0) = 0$ and $f_{FWM}(1) = 1$. Thus, fuzzy Boolean function of the GFWM filter is given by

$$f_{GFWM}(r^m(i)) = \begin{cases} \dfrac{[1+\exp\{-\alpha_1(1-\beta)\}]\cdot[1-\exp\{-\alpha_1(r^m(i)-\beta)\}]}{2[1-\exp\{-\alpha_1(1-\beta)\}]\cdot[1+\exp\{-\alpha_1(r^m(i)-\beta)\}]} + 0.5 & : \text{if } r^{(m)} > \beta \\ \dfrac{-[1+\exp(\alpha_2\beta)]\cdot[1-\exp\{-\alpha_2(r^m(i)-\beta)\}]}{2[1-\exp(\alpha_2\beta)]\cdot[1+\exp(-\alpha_2(r^m(i)-\beta)\}]} + 0.5 & : \text{otherwise} \end{cases} \tag{8}$$

When $\alpha_1 = \alpha_2 = \infty, \beta = 0.5$ (i.e., $f_{FWM}(\cdot) = f_{WM}(\cdot)$), the GFWM filter is equal to the conventional WM filter. On the other hand, if we set fuzzy Boolean function as $f_{FWM}(r^{(m)}(i)) = r^{(m)}(i)$, the FWM filter equal to the WA filter. If we set $\alpha_1 = \alpha_2 \leq 10, \beta = 0.5$ on $f_{GFWM}(r^{(m)}(i))$ of Eq.(8), the FWM filter almost equal to the WA filter. The FWM filter includes the WM filter and the WA filter. The FWM filter is suited for not only removing the impulse noise but also removing the Gaussian noise.

In this paper, the fuzzy Boolean function is defined by only three parameters. Figure 1 shows examples of fuzzy Boolean function. All weights of GFWM filters are decided independently. The structure of FWM filter is shown in Fig.2.

3.3 A design method of FWM filters

We attempt to design the FWM filter by means of LMS algorithm as follows :

$$\alpha_+(i) = \alpha_+(i-1) - \mu_1(\partial J(i)/\partial \alpha_+)$$
$$= \alpha_+(i-1) - \mu_1(y(i) - s(i)) \cdot \sum_{m=1}^{M-1} \frac{\partial f_{GFWM}(r^{(m)}(i))}{\partial \alpha_+} \tag{9}$$

where

$$\alpha_+ = \begin{cases} \alpha_1: \text{ if } r^{(m)} > \beta \\ \alpha_2: \text{ otherwise} \end{cases}$$

$$\begin{aligned}\beta(i) &= \beta(i-1) - \mu_2(\partial J(i)/\partial \beta) \\ &= \beta(i-1) - \mu_2(y(i)-s(i)) \cdot \sum_{m=1}^{M-1} \frac{\partial f_{GFWM}(r^{(m)}(i))}{\partial \beta} \quad (10)\end{aligned}$$

$$\begin{aligned}W_j(i) &= W_j(i-1) - \mu_3(\partial J(i)/\partial W_j) \\ &= W_j(i-1) - \mu_3(y(i)-s(i)) \cdot \sum_{m=1}^{M-1} \frac{\partial f_{GFWM}(r^{(m)}(i))}{\partial r^{(m)}(i)} \cdot \frac{\partial r^{(m)}(i)}{\partial W_j}\end{aligned} \quad (11)$$

where $J(i)$ is an error function which is defined as

$$J(i) = \frac{1}{2}\{y(i) - s(i)\}^2 \quad (12)$$

$s(i)$ is original signal of the training signal $x(i)$, $y(i)$ is output of the GFWM filter.

4 SIMULATION RESULT

In this section we present some experiment results illustrating the performance of the GFWM filter. All filter's window size set 5x5. The filter performance evaluates by MSE and PNSR.PNSR is defined by

$$PNSR = 10\log\frac{(255)^2}{MSE} \quad [\text{dB}] \quad (13)$$

In this experiment, the performance of FWM filters is compared with the performance of the optimal WM filter [5] and the Wiener filter. Each input image(original image "Lena") were used to train the three filters. Table 1 and 2 show the restoration results of the image "Lena" which is only degraded by Gaussian noise with standard deviation σ :10,20,30,and 40 and mixed noise (Gaussian noise with σ :0,20,30 and 40 + impulse noise 2%).Figure 3 shows the fuzzy Boolean functions given by training of using the images corrupted by mixed noise (Gaussain noise with σ :0,20,and40 + impulse noise 2%). These results show that the GFWM filter can change its properties form the WM filter to WA filter adaptable to the condition of the input image. Figure 4 shows filtered image.

Table 1 MSE results of Gaussian noisy images

	σ =10	σ =20	σ =30	σ =40
GFWM	42.8	99.6	155.7	205.2
WM	48.6	110.9	170.3	232.2
Wiener	53.8	116.6	169.7	218.2
SFWM	50.5	115.5	172.8	226.5

Table 2 MSE results of mixed noisy images

	σ =0	σ =10	σ =20	σ =30	σ =40
GFWM	19.4	52.2	107.4	163.7	224.1
WM	20.2	55.3	117.6	176.4	241.1
Wiener	117.3	130.5	160.9	199.4	243.4
SFWM	26.6	61.8	123.4	180.5	235.7

5 CONCLUSIONS

In this paper, one of fuzzy stack filter named the general type of fuzzy weighted median filter is proposed. A design method for FWM filter is given by using LMS algorithm. From some experimental result , we can see that the GFWM filter has properties of not only the WM filter but also the WA filter.

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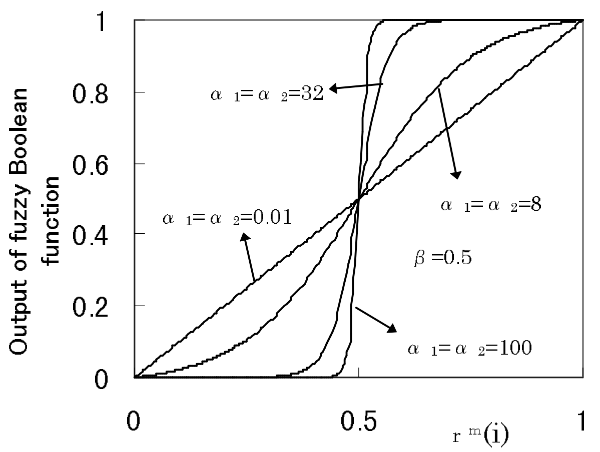

Fig.1 Fuzzy Boolean function

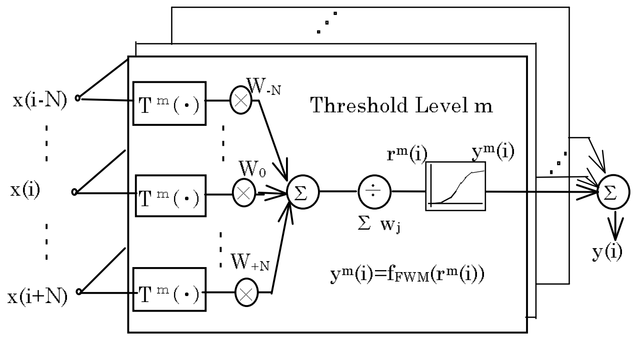

Fig.2 The fuzzy weighted median filter

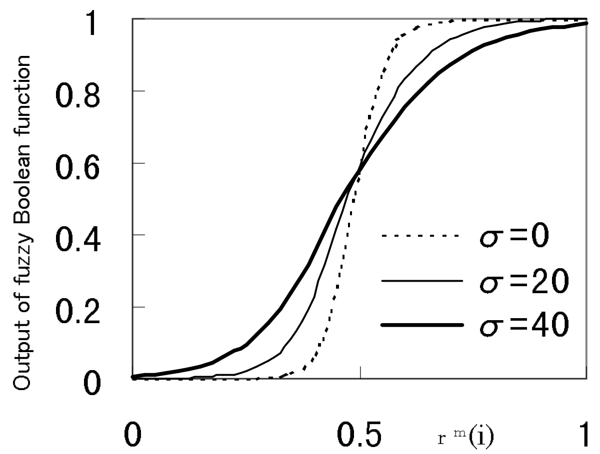

Fig.3 Fuzzy Boolean function tuned
by mix noisy images

Fig.4 (a) Noisy image
(Gaussian $\sigma=20$,Impulse 2%)

Fig.4 (b) GFWM filter

Fig.4 Filtered image(Zooming)