NONLINEAR CLONING TEMPLATE FOR ULTIMATE EROSION

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ABSTRACT

An image erosion architecture based on the CNN framework well suited for a VLSI implementation is presented. The erosion process can extract the center of gravity of an object coded by binary levels with a dynamical border peeling process. We first demonstrate that such an operator can not be implemented with the original model of CNN. Hence, the modification of the coupling function between cells is discussed. It is shown that an exclusive-or like gate (XOR) can be used to achieve a contour peeling process. The behavior of the new CNN model is investigated on a 1D chain of cells. The principle is then extended to a 2D map and is illustrated with a few numerical simulations. A reliable implementation approach of this CNN with its nonlinear cloning template is discussed in the appendix.

1. INTRODUCTION

Vision systems are gaining increasing attention on account of their potential in mobile systems control. These applications have clearly shown the interest of merging image sensing and processing on the same substrate leading to the concept of artificial retinas, also called smart retinas. Several smart retinas [1] have been reported since the pioneering work of R. F. Lyon on the optical mouse. Most implementations rely on analog processing rather than digital computing, thus taking benefit from the natural nonlinearities of silicium and a continuous time dynamics.

For target tracking or target marking applications, the on-chip extraction of the object's X-Y coordinates can allow high speed tracking and can be used for data fusion systems in conjunction with other signals. Our goal is to implement such an operator with the general purpose image processing grid array architecture of active elements, the so-called Cellular Neural Network model.

Cellular Neural Networks (CNN) have been the subject of a great interest since their introduction by Chua and Yang in 1988 [2]. Theoretical analysis as well as applications and implementations have been widely reported in the literature [3]. The key features of CNN are a discrete 2D topology together with a continuous time dynamics and a connection pattern reduced to the close vincinity. Furthermore, as the connection pattern is homogeneous among the whole network, its operating is controled only by a small set of weights, the so-called cloning template (CT), which is formulated with two matrices and one scalar (refer to equation (1) next section). Recent developments have proven the relevance of the CNN concept as a generic smart retina [4].

This paper presents a special class of CNN with a nonlinear CT which is intended to execute an ultimate erosion on a binary image. The erosion process is discussed in section II. It is shown that a CNN with a linear CT can not implement such an operator. Then, section III deals with the nonlinear coupling scheme. We show that an exclusive-or gate as the neihborhood feedback operator can play the role of the CT. The concept is illustrated with some numerical simulations. An elegant VLSI implementation of this CT in standard CMOS technology is then discussed in the appendix.

2. THE ULTIMATE EROSION OPERATOR

The object's location is easily taken from its barycenter which can be viewed as the result of an erosion process. This is similar as peeling the contour until a point remains alone as sketched in Fig. 1.

It should be noticed that our definition of the erosion is close to the ultimate erosion filter defined in morphological mathematics theory [5]. As a matter of fact, the ultimate erosion is the result of several elementary erosion passes which are stopped just before the removal of the last isolated pixels. An elementary erosion is usually expressed as:

$$Pixel_{i,j} = Min\left(\{Pixel_{k,l}, (k,l) \in N(i,j)\}\right)$$

where $N(i,j)$ is the set of pixels in the neighborood of the pixel (i,j).

In the remainder of this paper, the word erosion is taken in the sense of "ultimate erosion". Actually, this operator is close to a thinning or skeletization process, except that the resulting image is made up of isolated points rather than one-pixel-thick connected segments. A CNN for thinning images has already been presented [6]. However, an erosion CT can not be derived from this work due to two VLSI design considerations:

- The architecture is based on a multi-layer CNN. It uses height interconnected CNNs whereas the algorithm must stick to the 2D world of VLSI technology.

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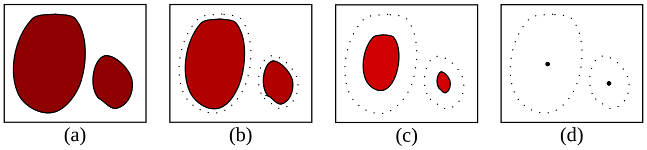

Figure 1: Principle of erosion: the object contours (a) are peeled (b-c) until a unique pixel remains (d). This point is the object's barycenter if the peeling process is isotropic.

- Moreover, the interaction radius is not unity for some planes of CNN. Once again, this is in conflict with the plane structure of an integrated circuit since 3D technologies[1] are not available on-the-shelf. Thus, long range signals needed by an interaction radius greather than one would lead to a significant silicon area lost by routing wires.

Starting from these facts, the erosion CT needs to be designed from scratch. A large panel of methods for synthetisis CT have already been reported. Among them, the transcription of pixel-based rules into a set of inequalities [7] is a promising approach for this application.

According to [2], the dynamics of a CNN's pixel is modeled as[2]:

$$\left\{ \begin{array}{lcl} \frac{dx_{ij}}{dt} & = & -x_{ij} + A \otimes y_{ij} + B \otimes u_{ij} + I \\ y_{ij} & = & f(x_{ij)} \end{array} \right. \quad (1)$$

where $f()$ is a piecewise linear threshold:

$$f(x) = \frac{1}{2}\left(|x+1| - |x-1|\right) \quad (2)$$

which codes the binary output levels as: white = -1 and black = +1. u_{ij} are the pixels of the image and the initial conditions are: $X(0) = U$. Equation (1) reflects a dynamical process which transforms a grey level image into a black and white one. The threshold function defines three domains, in which the dynamics is linear:

$$D_- : x < -1 \quad D_0 : x \in [-1, +1] \quad D_+ : x > 1$$

Taking into account the isotropy of the erosion process, the matrix of the CT should have the form of:

$$A = \begin{bmatrix} a_2 & a_1 & a_2 \\ a_1 & a_0 & a_1 \\ a_2 & a_1 & a_2 \end{bmatrix}, \quad B = \begin{bmatrix} b_2 & b_1 & b_2 \\ b_1 & b_0 & b_1 \\ b2 & b_1 & b_2 \end{bmatrix}$$

The objective is therefore to determine the CT which performs an ultimate erosion process. The set of possible solutions is the intersection of a list of inequalities drawn up with respect to a set of pixel-based rules. These rules include both the constraints of the initial conditions on the dynamics and of the ones that must be satisfied by the stationary states. Hence, the behavior of a single pixel with respect to its neighbors is given by:

- A black pixel surrounded by height other black ones must remain at the same state ($\dot{x}(0) \geq 0$ and $x(\infty) \geq 1$).
- In the same way, a white one should never be changed ($\dot{x}(0) \geq 0$ and $x(\infty) \geq 1$) since it belongs to the background.
- On the other side, if a black pixel belongs to a border, it must be turned white ($\dot{x}(0) < 0$ and $x(\infty) \leq -1$).
- At last, an isolated black pixel must not be switched since it is the desired final state ($x(\infty) \geq -1$).

Combining these rules together with (1) leads to an empty solution. Since the inequalities are built on linear relations, a CNN with linear CT can not implement an erosion filter. Thus, the erosion process can not be viewed as a linear separation problem.

3. THE NONLINEAR CT

Like the thinning problem [6], the erosion problem is not as easy as it looks. Instead of analysing the question with its two dimensions, it might be helpful to discuss the nonlinear CT on a one dimensional chain of coupled cells.

3.1. A 1D chain of coupled cells

A chain is a one dimensional structure of coupled cells. Cells interacts with their nearest neighbors. Then, an object can be viewed as a black segment among white cells. The erosion process transfoms this segment into a unique black point, which is the center of the segment if the action is symmetric.

The state dynamics of the i^{th} cell can be expressed as:

$$\frac{dx_i}{dt} = -x_i + ay_i + \gamma\left(x_{i-1}, x_{i+1}\right) \quad (3)$$

where y_i is the cell's output. Like in the CNN case, outputs are bounded by the threshold function $f()$ which is defined according to (2). Furthermore, the global stability is guaranteed if $a > 1$ [2, 3]. The interaction is characterized by the function $\gamma()$. Without any coupling, the cells should remain in one of the two stable states: black ($y = +1$) or white ($y = -1$).

The coupling effect can be defined by two simple rules:

- If a black point is surrounded by points of opposite color, its state must be switched to white ($\dot{x} < 0$).
- Otherwise, the point must remain the same ($\dot{x} = 0$).

The first rule stands for the peeling action. It is easy to see that, at the beginning of operations, only the ends of the segment are affected by this rule because they are inevitably surrounded by a black and a white one. If the action is repeated several times, the peeling is propagated from the ends to the middle of the segment. This process is similar to a cascade of dominos as sketched in Fig. 2.

[1]Even though 3D technologies are a growing field of interest, they are not yet proposed by multi-project fundry services for research purposes. These technologies, based on stacked SOI layers obtained through the recrystallization of poly-Si by an energetic beam, need state-of-the-art high cost fabrication processes.

[2]Here $\otimes$ denotes a two-dimensional correlation product at the pixel (i, j): $Z \otimes w_{ij} = \sum z_{k,l} w_{k+i,l+j}$. This model is expressed unitless, ie: $R = C = 1$in [2, 3].

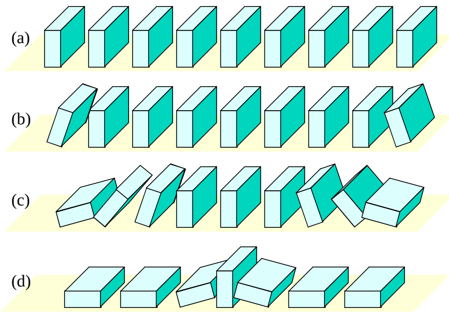

Figure 2: The erosion principle is based on a cascade of dominos. (a): chain of standing dominos. (b): the chain is perturbated at its ends, the dominos are pushed towards their neighbors. (c): the dominos fall is propagated step by step. (d): the process stops as soon as a unique domino remains upright. The principle is here depicted for the case of a chain made up of an odd set of dominos. It can be easely extend to the case of an even set, the two middle dominos would stand upright at the end of the operations.

x_{i-1}	x_{i+1}	$\frac{dx_i}{dt}$
white	white	0
white	black	<0
black	white	<0
black	black	0

Table 1:

This principle applied to the i^{th} cell is condensed in table 1. As it can be seen, the coupling function follows a kind of exclusive-or (XOR) behavior. A design with a standard CMOS technology should certainly take benefit from this property.

The $\gamma()$ function can now be expressed. First of all, it must be symmetric, ie: $\gamma(l,r) = \gamma(r,l)$. Furthermore, the function can be expressed with respect to the sum of its two variables. Indeed, this function needs only two parameters:

$$\gamma(l,r) = \begin{cases} 0, & |l+r| > T \\ -s, & |l+r| < T \end{cases} \tag{4}$$

with s the negative step level and T the trigger's threshold.

In order to prove the correct operating of this new model and to bound the paramaters, a short analysis of the model is carried out.

3.1.1. Equilibrium points

As in the standard CNN model, a cell described by (3) has three equilibrium states according to the operating point of (2). Only x_{Q-} and x_{Q+} are stable while x_{Q0} is unstable. Of course, $a > 1$ is a necessary condition to ensure global stability and convergence of the system.

$$x_Q = \begin{cases} x_{Q+} = a + \gamma & x_Q > 1 \\ x_{Q0} = \frac{\gamma}{1-a} & x_Q \in [-1;1] \\ x_{Q-} = -a + \gamma & x_Q < -1 \end{cases} \tag{5}$$

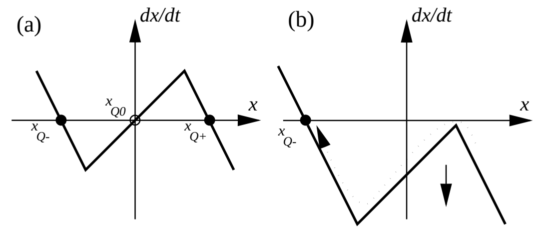

Figure 3: Pixel inversion in the plane $(x, \dot{x})$. Starting from black (a), the negative step imposed by $\gamma()$ pushed the dynamic route downward, thus vanishing the black resting point (b). The route is then open to the white resting point, which has slightly moved on the left.

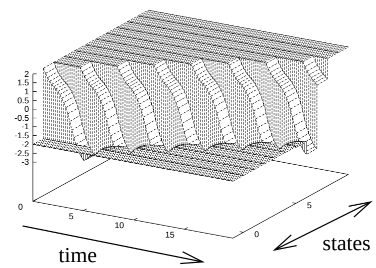

Figure 4: Propagation of pixel inversion from front-left to right ($x_i(t)$). The chain's state is drawn along the y-axis while the time is drawn along the x axis.

3.1.2. Pixel switching

The only way for a black pixel to switch off is to be surrounded by a black one and a white one. Even though the system is modeled by a piecewise linear differential equation, the expression of the state vector with respect to time is not an easy problem and would lead to a bulky time-series. In order to illustrate the switching principle, the discussion is here restricted to one cell with two static neighbors: a black and a white one. Then, starting from $x(0) = x_{Q+} = a$, its dynamic is as follows:

$$\frac{dx}{dt} = -x + a.f(x) - s$$

The action of $\gamma()$ persists since this cell is surrounded by two static pixels. The evolution of $x(t)$ is made up of exponential branches.

If the negative step $(-s)$ is sufficient, the two rightmost equilibrium points disappear. The only remaining one is the leftmost one, which is < -1, thus representing a white point. This operation is depicted on Fig. 3. It is easy to understand that the lower bound of s is:

$$s_{min} = a - 1 \tag{6}$$

in order to garantee that x_{Q-} is the only state.

3.1.3. Propagation phase

The switching mechanism has been depicted on an isolated cell. For a chain of cells, the pixel inversion gives birth to an imbalance which excites the next black cell and in turn switches it to white. This perturbation is then propagated step by step on each neighboring black cell. The behavior of the cascade of dominos is found again.

The negative step of $\gamma()$ on the i^{th} cell persists until $|x_{i-1}+x_{i+1}|<T$. Before the next switch is completed, $|x_{i-1}+x_{i+1}|<T$ is false and the perturbation stops. Figure 4 illustrates this mechanism (the left neighbor of the first cell is a static white point).

3.1.4. Stopping phase

Before detailing the stopping procedure, it must be pointed out that the case of an odd chain and an even one must be investigated separately. Indeed, while an odd chain would lead to a unique black pixel at the end of the erosion process, an even one would lead to two black pixels. Furthermore, it should be noticed that the equilibrium points defined by (5) here define the equations of the null-clines of each cells ($\dot{x}_1=0$ and $\dot{x}_2=0$) [8].

The odd chain: the study of the stopping phase for an odd chain is carried out on a three cells structure which are enclosed with two static white pixels to bring the imbalance ($x=x_{Q-}=-a$) as depicted on Fig. 5a. Thanks to a symmetry property ($x_1=x_3$), the system can be modeled by:

$$\begin{cases} \frac{dx_1}{dt}=-x_1+a.f(x_1)+\gamma(-a+x_2) \\ \frac{dx_2}{dt}=-x_2+a.f(x_2)+\gamma(2x_1) \end{cases}$$

The two null-clines are defined by (5) except when the coupling function is activated, *ie*: $\gamma(l,r)=-s$. In such a case, the only remaining equilibrium point is: $x_{Q-}=-a-s$ if $s>s_{min}$. This appears for the first cell while:

$$x_2\in[a-T,a+T]$$

and for the second one while:

$$x_1\in\left[-\frac{T}{2},\frac{T}{2}\right]$$

Starting from $X(0)=(x_{Q+},x_{Q+})=(a,a)$, the trajectory must follow a pathway to reach a resting state which keeps $X(\infty)=(white,black)=(<-1,>+1)$. This pathway is constraint by the null-clines for the two states. This is best illustrated in the plane (x_1,x_2) shown in Fig. 5a. Because its neighbors have an opposite state, the cell x_1 is kept under the influence of $\gamma()$, *ie*: $x_{Q1}=-a-s$ is the only possible resting state and $dx_1/dt<0$. This holds as long as x_2 lies in $[a-T;a+T]$. On the opposite, the null-cline $\dot{x_2}=0$ is still defined by (5) except in the domain $x_1\in[-T/2;T/2]$ where the only remaining equilibrium point is also $x_{Q2}=-a-s$. As $x_2(0)=x_{Q+}$, it stays at rest until the first cell enter in $[-T/2;+T/2]$. Then, $x_2(t)$ begins to decrease (the coupling effect is activated for the second cell). As soon as $x_1(t)<-T/2$, the dynamics of the first cell is attracted by $x_{Q+}=+a$. Thus, the final state is: $X_Q=\{-a-s,+a\}$ as previously asked for.

The even chain: the stop of the processing is investigated on a chain of four cells enclosed by two static white pixels (Fig. 5b). As in the odd case, the model is reduced

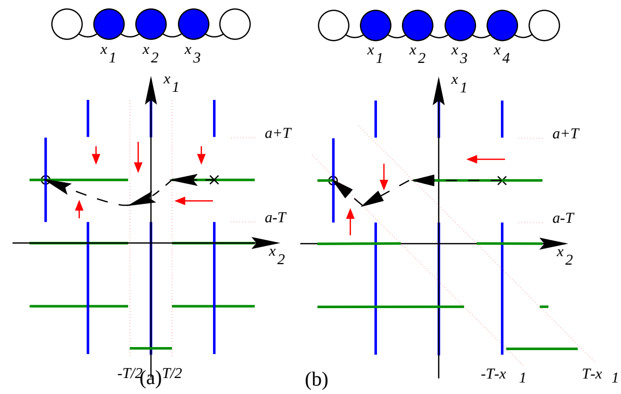

Figure 5: State plane (x_1x_2) for an odd chain made up of 3 cells (a) and for an even chain made up 4 cells (b). Null-clines are drawn with solid lines while the trajectory is a dashed line. Boundaries of the basin of influence of the coupling function are drawn with dotted lines. The orientation of the velocity field of x_1 or x_2 is shown by the arrows according to their orientation.

to a two degrees of freedom system thanks to symmetry considerations ($x_1=x_4$, $x_2=x_3$):

$$\begin{cases} \frac{dx_1}{dt}=-x_1+a.f(x_1)+\gamma(-a+x_2) \\ \frac{dx_2}{dt}=-x_2+a.f(x_2)+\gamma(x_1+x_2) \end{cases}$$

The assumptions are the same as before. The starting state is $X(0)=(a,a)$ and the resting one must keep $X(\infty)=(white,black)$ valid. Once again, the trajectory is guided by the pathway constraint by the null-clines. In the even case, the x_1-null-cline is the same as for the odd chain whereas the x_2 one is defined by $x_2=af(x_2)+\gamma(x_1+x_2)$. Thus, for the cell x_2, the coupling function is activated while:

$$x_2\in[-T-x_1;T-x_1]$$

This leads to a diagonal region in which $\dot{x_2}<0$ as sketched in Fig. 5b. If we want the final state to be the same as before, the line $-T-x_1$ must lie above the resting point $(-a-s,a)$ which yields:

$$T<s$$

3.1.5. Numerical simulations

Figure 6 reports two simulations of a chain made up of an even and an odd number of cells initially excited by a black pixel and enclosed with a small numbers of white ones.

The erosion process is clearly shown starting at the edges of the black segment and peeling it until a unique black pixel remains in the odd case (two pixels in the even case). Of course, the transient period length is a function of the size of the initial segment.

3.2. Generalization to a 2D map

The peeling principle has been introduced on a 1D chain. Extending it to a 2D map leads to this model of CNN with a nonlinear CT:

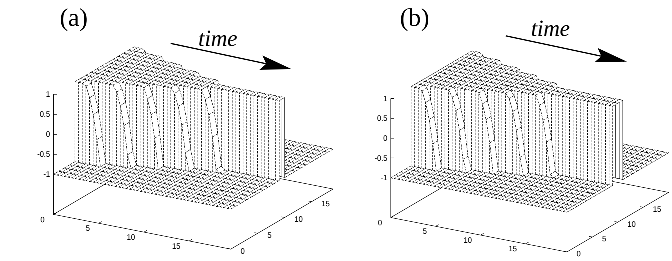

Figure 6: cells' output. (a): 11 black cells and 4 whites at each side of the segment. (b): the initial state is made up of 12 black pixels and the same number of white one.

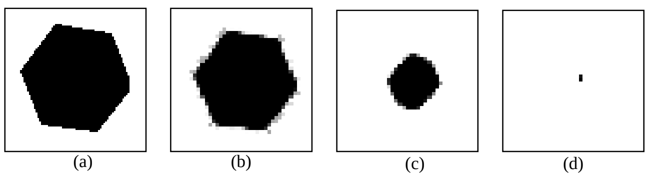

Figure 7: Ultimate erosion on an arbitrary shape. (a): initial image, (b): after 1 time unit (tu), (c): after 10 tu, (d): final state after 20 tu.

$$\left\{ \begin{array}{rcl} \frac{dx_{i,j}}{dt} & = & -x_{i,j} + a\left[\gamma\left(x_{i-1,j}, x_{i+1,j}\right) + \gamma\left(x_{i,j-1}, x_{i,j+1}\right)\right] \\ y_{i,j} & = & f\left(x_{i,j}\right) \end{array} \right. \quad (7)$$

where $f(x)$ is defined as (2). A simulation is depicted on fig. 7.

It should be noticed that (7) involves a 4-neighborood CT.

4. CONCLUSION

An new Cellular Neural Network nonlinear CT for binary image erosion has been presented. Its operating has been demonstrated for a one dimensionnal chain of coupled cells. Its capabilities have been illustrated through numerical simulations in the two dimensional case of images. Its reliability for a VLSI implementation in standard CMOS technology will be briefly pointed out in the appendix.

In order to decrease the area per cell, this nonlinear scheme can take benefit from the Full Range Signal (FSR) model developed by Espejo *et al.* [9].

A forthcoming work would be to design another nonlinear CT for image skeletization using the same approach.

5. APPENDIX: IMPLEMENTATION OF THE XOR

The coupling function has been identified as an exclusive-or gate (XOR). However, unlike the usual XOR CMOS gate, where input and ouput signal are voltages, our operator must be current output.

Such a device can be derived from two CMOS inverters with their output ports shorted. Remember that a current

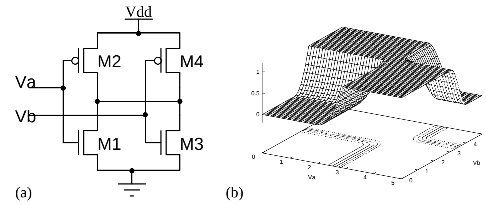

Figure 8: (a) Principle: an XOR cell with voltage input and current output. (b) The output is the sum of the two drain currents (M1+M3).

flows through a CMOS inverter gate while switching, it is easy to understand the operating reported on Fig 8. If V_a and V_b share the same state (high or low), the two inverters (M1-M2 and M3-M4) are off, thus no current can flow. However, if V_a and V_b are in two complementary states, the two opposite transistors are open, allowing a current to flow from V_{DD} to GND:

- If V_a is high and V_b is low, the current sinks through M1-M4.
- If V_a is low and V_b is high, the current goes through M2-M3.

A SPICE simulation of a complete cell is reported on Fig. 8b.

6. ACKNOWLEDGEMENT

L. M. is indebted to Dr. L. Borne for his introduction to the fascinating world of morphological mathematics. The authors wish to thank A. Koneke and A. L. Schneider for our fruitful discussions and J-L. Urban for carrefully reading the manuscript.

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