MEASURING THE LINEAR AND NONLINEAR PROPERTIES OF ELECTRO-ACOUSTIC TRANSMISSION SYSTEMS

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ABSTRACT

Competitive audio consumer products require not only cheap signal processing hardware but also low-cost analog equipment and sound transducers. The nonlinear distortions produced by these electro-acoustic transmission systems cannot be described and analyzed by standard methods based on linear systems theory alone. In order to take the nonlinear properties into account, we present a measurement method for the linear and nonlinear transmission characteristics of almost arbitrary systems and show its application to the analysis of electro-acoustic systems. Examples demonstrate the measurement of the impulse response of a loudspeaker-enclosure-microphone-system with cheap analog equipment.

1 INTRODUCTION

Real-time signal processing used to be the most expensive part of acoustic echo and noise control applications. The additional cost of complementing the digital hardware with good quality audio equipment was tolerable and consequently, nonlinear effects of sound transducers could be neglected.

The situation is changing with the availability of cheap computing power for real-time applications. Competitive audio consumer products require not only cheap signal processing hardware but also low-cost sound transducers. Software-only solutions of speech communication features for desktop computers have to rely on built-in microphones and speakers, whatever their quality may be. Therefore, nonlinear distortions have to be taken into account in the design of low-cost electro-acoustic systems.

But also the digital transmission of speech or audio signals is subject to nonlinear effects. Subband-coding with the least possible number of bits assigned to each band is a standard technique. Also low-cost analog-digital-converters produce distortions, which cannot be described by linear effects only.

Many acoustic echo and noise control applications require to measure or estimate the properties of the loudspeaker-enclosure-microphone-system (LEMS) including any digital pre- and post-processing. The nonlinear effects described above may show up in many places of this electro-acoustic transmission chain, so that it is not always possible to consider them by a proper theoretical analysis. Also measurements of the distortion factor of isolated components do not give a complete picture of the nonlinear behaviour of the overall system. On the other hand, common measurement methods for the room impulse response record only the linear transmission characterics and are blind for nonlinearities.

This contribution presents a measurement method for the linear and nonlinear transmission characteristics of almost arbitrary systems and shows its application to the analysis of electro-acoustic systems. The theoretical foundations of the method are described. Examples for the measurement of a room impulse response with cheap sound transducers show how nonlinear effects can be detected and evaluated.

2 PROBLEM DESCRIPTION

Fig. 1 shows an electro-acoustic transmission system for the measurement of an LEMS, consisting of amplifiers, digital-to-analog and analog-to-digital converters. A discrete measurement system MS excites the transmission system with a discrete input signal $v(k)$ and records its response $y(k)$. The design of the measurement system is the topic of this contribution.

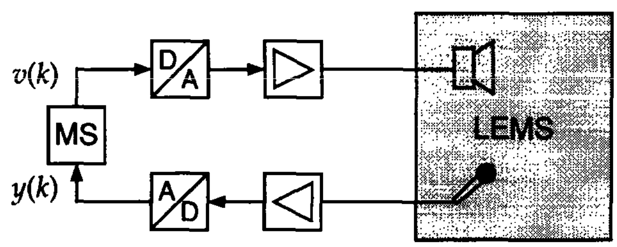

Figure 1: Electro-acoustic transmission system for the measurement of a loudspeaker-enclosure-microphone-system (LEMS)

3 LINEAR APPROXIMATION

The transmission of acoustic signals from the loudspeaker to the microphone is described by the linear wave equation [5]. However, if other sound sources are present in the LEMS, the measurement signal may be superimposed by additive background noise. Amplifiers and sound transducers can be modelled as linear systems, if the distortions produced by them are negligible. This assumption is only valid for good quality (expensive) equipment. The converters are nonlinear systems by their very nature. They can be approximated by linear systems only for a high wordlength of the digital signals.

If background noise, distortions of the analog equipment, and quantization effects are neglected, the electro-acoustic system from fig. 1 can be replaced by a linear model with an overall impulse response $h_{\text{lin}}(k)$ between input $v(k)$ and output $y(k)$ (see fig. 2).

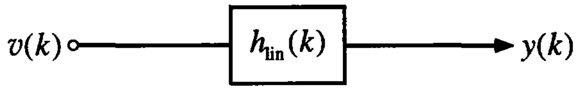

Figure 2: Linear model

An estimation $\hat{h}_{\text{lin}}(k)$ of this impulse response can be obtained with the Discrete Fourier Transformation (DFT), e.g. in the most simple case as

$$\hat{h}_{\text{lin}} = \text{DFT}_M^{-1}\left\{\frac{\text{DFT}_M\{y(k)\}}{\text{DFT}_M\{v(k)\}}\right\} \tag{1}$$

A number of techniques exist, which differ in the choice of the input signal $v(k)$, the DFT length M, and in more elaborate details of obtaining the estimate $\hat{h}_{\text{lin}}(k)$ [4]. However, all these techniques rely on the assumption of linearity and are blind for any of the nonlinear effects described above.

4 WEAKLY NONLINEAR APPROXIMATION

A detailed account of all possible nonlinear contributions in fig. 1 leads to a general nonlinear model according to fig. 3. Due to the variety and different nature of the nonlinear components, an exact nonlinear model **S** of the transmission system from fig. 1 would be too complex to be handled efficiently.

Figure 3: Nonlinear model

In order to circumvent this difficulty, we approximate the nonlinear system **S** by a weakly nonlinear model as shown in fig. 4. It consists of a parallel arrangement of a linear system $\mathbf{S}_{\text{L}}$ and a nonlinear system $\mathbf{S}_{\text{N}}$.

Again, it is not feasible to derive $\mathbf{S}_{\text{L}}$ and $\mathbf{S}_{\text{N}}$ analytically from the components of the transmission system. Instead, the characteristics of both systems

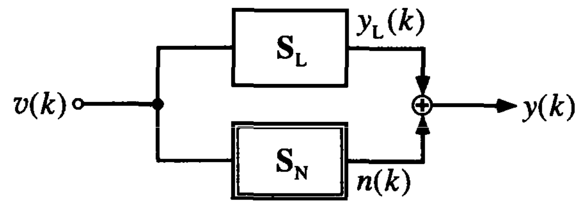

Figure 4: Weakly nonlinear model

have to be obtained experimentally from suitably chosen input signals $v(k)$ and the corresponding responses $y(k)$.

A method for the determination of $\mathbf{S}_{\text{L}}$ and $\mathbf{S}_{\text{N}}$ from measurements of $v(k)$ and $y(k)$ is described in the following. It is based on the assumption that the impulse response of the linear subsystem does not depend on the variance of the input signal. In this case, the arrangement of fig. 4 is called a *weakly nonlinear model.*

This kind of nonlinear systems analysis has been succeessfully applied to the measurement of implemented digital systems, i.e. the simultaneous determination of the frequency response and the quantization noise in digital filters and filter banks [3, 2]. Here, it will be applied to the mixed analog-discrete electro-acoustic system described above.

5 MEASUREMENT METHOD

5.1 Frequency response of the linear subsystem $\mathbf{S}_{\text{L}}$

The frequency response of the linear subsystem $\mathbf{S}_{\text{L}}$ is determined such that the contribution of the nonlinear subsystem $\mathbf{S}_{\text{N}}$ to the total output signal $y(k)$ becomes a minimum. This is achieved by expressing the output $y_{\text{L}}(k)$ of $\mathbf{S}_{\text{L}}$ in fig. 4 by a convolution with the impulse response $h(k)$ of $\mathbf{S}_{\text{L}}$: $y_{\text{L}}(k) = h(k) * v(k)$. The minimization of the quadratic mean value of the output of the nonlinear system $\epsilon(k) = \mathcal{E}\left\{\mid n(k)\mid^2\right\}$ is equivalent to determining the impulse response $h(k)$ such that the expected value $\mathcal{E}\left\{\cdot\right\}$

$$\epsilon(k) = \mathcal{E}\left\{\mid y(k) - h(k) * v(k)\mid^2\right\} \tag{2}$$

becomes a minimum. After some calculations, similar to the derivation of the Wiener-filter, we obtain the frequency response as

$$H(e^{j\Omega}) = \sum_{k=-\infty}^{\infty} h(k)e^{-j\Omega k} = \frac{\Phi_{yv}(e^{j\Omega})}{\Phi_{vv}(e^{j\Omega})}\,. \tag{3}$$

$\Phi_{vv}(e^{j\Omega})$ is the power density spectrum (PDS) of the input $v(k)$ and $\Phi_{yv}(e^{j\Omega})$ is the cross power density between output $y(k)$ and input $v(k)$.

Since these power densities are not known in advance, they have to be determined from the signals

$v(k)$ and $y(k)$. In general, a wide range of possible input signals and a variety of corresponding spectral estimation methods could be utilized. Here, we will focus on periodic signals $v(k)$ and $y(k)$ with period M. Then it is sufficient to determine $H(e^{j\Omega})$ at M discrete frequencies $\Omega_\mu = \mu 2\pi/M, \mu = 0, \cdots, M-1$. The values $H(e^{j\Omega_\mu})$ of (3) can now be expressed by $V(\mu) = \mathrm{DFT}_M\{v(k)\}$ and $Y(\mu) = \mathrm{DFT}_M\{y(k)\}$ [1]

$$H(e^{j\Omega_\mu}) = \frac{\mathcal{E}\left\{Y(\mu)V^*(\mu)\right\}}{\mathcal{E}\left\{|V(\mu)|^2\right\}}, \ \mu = 0, \cdots, M-1 \ . \quad (4)$$

The expected values $\mathcal{E}\{\cdot\}$ are calculated from the measured input and output sequences by applying L sample sequences $v_\lambda(k)$, $\lambda = 0, \ldots, L-1$ of the same stochastic process to the transmission system and by averaging the results. Thus, the estimated value $\hat{H}(e^{j\Omega_\mu})$ for the frequency response of the linear subsystem is obtained

$$\hat{H}(e^{j\Omega_\mu}) = \frac{\sum\limits_{\lambda=0}^{L-1} Y_\lambda(\mu)V_\lambda^*(\mu)}{\sum\limits_{\lambda=0}^{L-1} |V_\lambda(\mu)|^2}, \ \mu = 0, \cdots, M-1 \ . \quad (5)$$

The procedure is simplified, if the DFT-spectrum of the sample sequences has a constant magnitude $|V_\lambda(\mu)| = |V(\mu)|$. This can be achieved by deriving the sample sequences $v_\lambda(k)$ from an arbitrary periodic, deterministic signal $v(k)$ by adding a stochastic phase term

$$v_\lambda(k) = v_\lambda(k+M) = v(k)e^{j\varphi_\lambda} \ . \quad (6)$$

The phase φ_λ is a stochastic variable, equally distributed in the interval $[-\pi, +\pi)$.

For input signals of this kind, the computation of the estimated value simplifies to

$$\hat{H}(e^{j\Omega_\mu}) = \frac{1}{LV(\mu)}\mathrm{DFT}_M\left\{\sum_{\lambda=0}^{L-1} y_\lambda(k)e^{-j\varphi_\lambda}\right\}. \quad (7)$$

Fig. 5 shows the structure of the resulting measurement arrangement. In our application, a complex chirp signal

$$v(k) = e^{j(\pi/M)k^2} \quad (8)$$

has been used as deterministic component $v(k)$ of the input. This signal achieves the minimum possible crest factor.

5.2 Power spectral density at the output of the nonlinear subsystem $\mathbf{S_N}$

Once the frequency response of the linear subsystem $\mathbf{S_L}$ is estimated, we can use this result to obtain the power spectral density $\Phi_{nn}(e^{j\Omega_\mu})$ of the nonlinear

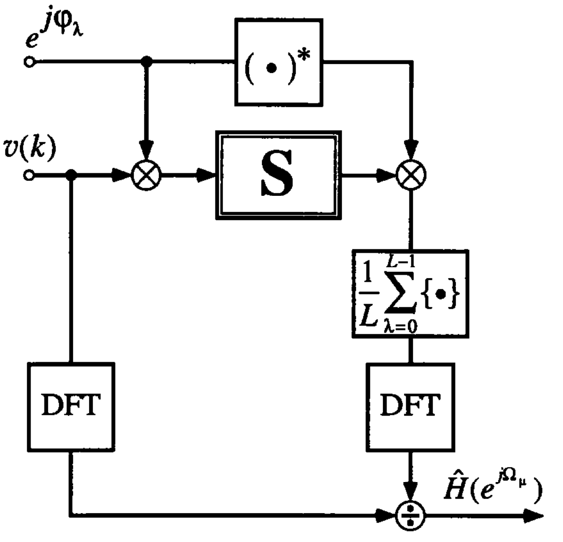

Figure 5: Measurement arrangement for the frequency response of the linear subsystem $\mathbf{S_L}$. $(\bullet)^*$ denotes the conjugate value.

subsystem $\mathbf{S_N}$. From fig. 4 follows for the estimated output of the nonlinear system

$$\hat{n}(k) = y(k) - \hat{h}(k) * v(k) \ , \quad (9)$$

where $\hat{h}(k)$ is the estimated impulse response of $\mathbf{S_L}$. The estimation of $\Phi_{nn}(e^{j\Omega_\mu})$ is performed in the frequency domain by evaluation of

$$\hat{\Phi}_{nn}(e^{j\Omega_\mu}) = \quad (10)$$
$$= \frac{1}{M(L-1)}\sum_{\lambda=0}^{L-1}\left[|Y_\lambda(\mu)|^2 - \left|\hat{H}(e^{j\Omega_\mu})\right|^2 |V_\lambda(\mu)|^2\right]$$

6 RESULTS

This section presents measurements of an LEMS both with high quality and low quality equipment. The LEMS was an anechoic chamber containing the measurement gear which caused some reflections. The high quality equipment was a studio microphone, a preamplifier (Bryston Mod. PB4) and an active loudspeaker (GENELEC Mod. 1013A). For the low quality measurements, preamplifier and loudspeaker were replaced by a one-chip amplifier (TBA 820M) with a cardboard mounted noname speaker (6 cm diameter).

Fig. 6 shows the frequency response of the LEMS, measured with high quality audio equipment. Here, we used a standard measurement procedure according to (1) which assumes a strictly linear system behaviour.

Fig. 7 shows the same measurement using low-cost equipment and again a standard measurement method. Due to this equipment a different frequency

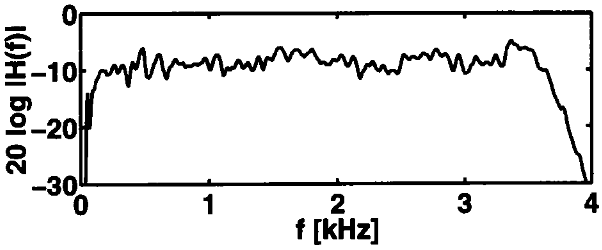

Figure 6: Frequency response using high quality audio equipment and standard measurement method

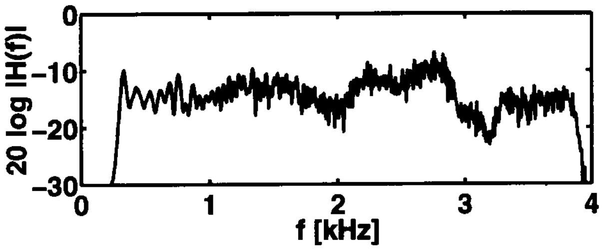

Figure 7: Frequency response using low-cost audio equipment (standard measurement method)

response has to be expected although the room impulse response was not affected by the change. Nevertheless, the measured frequency response shows an unexpected strong noise-like behaviour. This is due to the fact that conventional measurement procedures are not able to distinguish between linear and nonlinear system components, the latter being caused by distortions of the amplifier and speaker. Thus, the result is a superposition of both components.

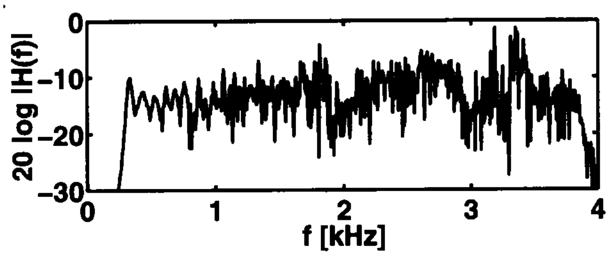

Figure 8: Frequency response using low-cost audio equipment with background noise (standard measurement method)

The frequency response measurement shown in Fig. 8 was performed with an additional broadband background noise 4dB below the signal level. It was produced by a noise generator fed into a guitar amplifier. The detrimental distortion of the measurement result by the superimposed noise source is obvious.

In contrast, Fig. 9 shows the measurement results with our proposed method for the low-cost equipment with background noise as in Fig. 8. This method is capable of recording linear and nonlinear effects separately. The method yields the frequency response of the linear subsystem as well as the PDS of nonlinear distortions including noise. Obviously, the true frequency response deviates from the original one (Fig. 6) due to additional linear distortions of the low-cost equipment. Thus, our method is an efficient way to measure the true response even in the presence of nonlinear distortions and background noise. Moreover, also the nonlinear system components and noise characteristics can be identified and described by the PDS of the disturbances.

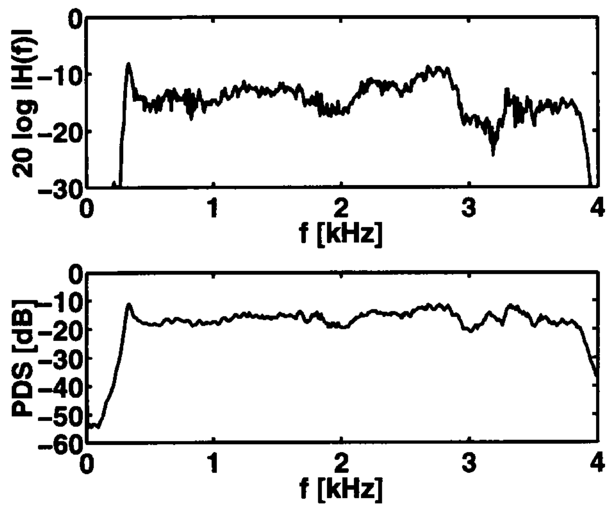

Figure 9: Frequency response and power density spectrum using low-cost audio equipment with background noise (proposed measurement method)

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