Array Signal Processing in the Presence of Unknown Noise Fields

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ABSTRACT

In the context of the narrow-band or wideband array processing problem, in this paper we develop a robust algorithm to improve the accuracy of the estimation of the direction of arrival of the sources. It is well known that when the noise cross-spectral matrix is unknown, these estimates may be grossly inaccurate. Using both a propagation operator and noneigenvector algorithm to estimate the noise subspace projection matrix we develop a new robust algorithm for the source characterisation problem in the presence of noise with an unknown cross-spectral matrix. When shall show that the performance of bearing estimation algorithms improves substantially when our robust algorithm is used. Simulation results are presented for the band noise spectral matrix.

1. INTRODUCTION

The estimation of directions of arrival of multiple narrow-band or wideband sources in the noise is a classic problem in array signal processing[1-6]. Many bearing estimation procedures have been reported in the literature, among which the various eigenanalysis-based methods have been the focus of many studies[1-5]. The eigenstructure procedure is computationally costly when the number of sensors is large. These methods are also dependent on the structure of the noise cross-spectral matrix. A fundamental assumption for most direction finding algorithms, developed in the last decade, is that the noise is spatially and temporally white or the spatial correlation structure of the background noise is known to within a multiplicative scalar. Then, the localisation algorithm can be usually modified in a straightforward manner to include it in the treatment. In practice, this assumption is rarely fulfilled. This is due to the fact that the noise has several origins, such as traffic noise, ambient sea noise, or flow noise, and sometimes the source signals with low power or undetected are assimilated to the noise, which are often spatially correlated. In recent years, there has been a growing interest in the problem of improving high resolution eigenstructure techniques with objective of lowering the signal to noise ratio resolution threshold or the spatially colored noise[7-13]. The ambient noise is unknown in practice, therefore its modelisation or its estimation is necessary. The methods developed for this problem are very few and there is not a definitive solution to this problem. There are some practical methods : in[11] two methods are obtained by optimisation of criterion and by using AR or ARMA modelling of noise. In[12-13] the spatial correlation matrix of noise is modelled by the known Bessel functions. As in[8] the ambient noise spectral matrix is modelled by a sum of hermitian matrices known up to multiplicative scalar. In[14] this estimate is obtained by measuring the array cross-spectral matrix when no signals are present. This procedure assume that the noise is not varying with time, which is not fulfilled in several domain applications. Another possibility[15] arises when the correlation structure is known to be invariant under a translation or rotation. The so-called

covariance differencing technique can be then applied to reduce the noise influence. In this method, two identical translated and/or rotated measurements of the array cross-spectral matrix are required and hypothesises the invariance of the noise cross-spectral matrix, while the source signals change between the two measurements. The estimate of the noise cross-spectral matrix is eliminated by simple subtraction. Furthermore, this method cannot be applied when the source cross-spectral matrix satisfies the same invariance property or when only one measurement is available. In[8-9] a particular modelling structure noise spectral matrix, which takes into account the characteristic noise relative to its origins, is given. In general, even if the individual treatments is different in these articles, the obtained noise structure matrix is the same.
In this paper the band noise spectral matrix, the classical noise model, is used . By means of a linear operator and an iterative algorithm for estimating the spatial correlation length, a new estimator for the direction of arrival of the sources is developed. The proposed algorithm is based upon a particular partition of the received signal vector which leads one to obtain an approximation of the noise subspace matrix without eigendecomposition.

2. PROBLEM FORMULATION

Consider an uniform linear array composed of N identical sensors separated from each other by a distance d. Let P, (P<N), sources impinge on the array from the directions $\{\theta_1, \theta_2, \theta_3, \ldots, \theta_p\}$. The signal received at the ith sensor can be expressed as : $r_i(t) = \sum_{p=1}^{P} s_p(t-\tau_{ip}) + n_i(t), i=1,\ldots,N$ (1)
where $n_i(t)$ is the additive noise at the ith sensor, $s_p(t)$ is the signal emitted by the pth source and τ_{ip} is the propagation delay associated with the pth source and the ith sensor. Rewriting (1) in matrix notation, in the frequency domain, we obtain :

$$\mathbf{r}(f_j) = \mathbf{A}(f_j)\mathbf{s}(f_j) + \mathbf{n}(f_j),\ j=1,\ldots,M$$

(2)

where $A(f_j)$ is the NxP transfer matrix of the source-sensor array systems with respect to some chosen reference point, $A(f_j) = [\mathbf{a}(f_j, \theta_1), \mathbf{a}(f_j, \theta_2), \ldots, \mathbf{a}(f_j, \theta_p)]$
$\mathbf{a}(f_j, \theta_i)$ is the steering vector of the array toward the direction, θ_i at the frequency fj.
Assume that the signals and the additive noises are uncorrelated, and the noises are assumed to be partially correlated between the sensors. It follows from these assumptions that the spatial cross-spectral matrix of the observation vector at frequency fj is given by :

$$\Gamma(f_j) = E[\mathbf{r}(f_j)\mathbf{r}^+(f_j)]$$

using the above assumptions, we obtain :

$$\Gamma(f_j) = \mathbf{A}(f_j)\Gamma_s(f_j)\mathbf{A}^+(f_j) + \Gamma_n(f_j)$$

where the (i, k)th noise spectral matrix element $\Gamma_n^{(i,k)}(f_j) = 0 \quad \forall |i-k| > L$. $\Gamma_n(fj)$ is a banded positive-definite matrix with bandwidth or spatial correlation length L.

3. NOISE SUBSPACE ESTIMATION WITHOUT EIGENDECOMPOSITION

In this section, a noneigenvector algorithm is developed to estimate the noise subspace used in the high resolution methods. The basic idea is that the source spectral matrix is an order N but is of rank P. Therefore, there exists (N-P) rows of matrix A(fj) linearly dependent of P others rows. Assume that the number of sensors is such that N> L + 2P, then we have :

$$\mathbf{A}^+(f_j) = [\underbrace{\mathbf{A}_1^+(f_j)}_{P}\ \underbrace{\mathbf{A}_2^+(f_j)}_{K}\ \underbrace{\mathbf{A}_3^+(f_j)}_{P}\ \underbrace{\mathbf{A}_4^+(f_j)}_{1}]$$

where K = N- 2P-1. The two block matrices $A_1(fj)$ and $A_3(fj)$ are assumed non singular. This assumption can be always ensured with any re-arranging sensor order. In plane waves case and linear antenna A(fj) is a Vandermonde matrix.
Let $\Pi(fj)$ the matrix which lies the dependency existing between the rows of $A_3(fj)$ and $A_4(fj)$.
$A_4(fj) = \Pi^+(fj)\, A_3(fj)$
It is to see that the two last block matrices of the first row of $\Gamma(fj)$ are :

$$\Gamma_{13}(f_j)=\mathbf{A}_1(f_j)\Gamma_s(f_j)\mathbf{A}_3^+(f_j)$$

$$\Gamma_{14}(f_j)=\mathbf{A}_1(f_j)\Gamma_s(f_j)\mathbf{A}_4^+(f_j)$$

or,

$$\Gamma_{14}(f_j)=A_1(f_j)\Gamma_s(f_j)A_3^+(f_j)\Pi(f_j)$$

then $\Pi(fj)$ is estimated by, $\Pi(f_j)=(\Gamma_{13}(f_j))^{-1}\,\Gamma_{14}(f_j)$ is the propagation operator. Now, since the linear property is fulfilled an orthogonal matrix to A(fj) can be constructed, let the (2P+K+1)x1 vector **q**(fj) define as(using the same partition as for the matrix **A**(fj)) :

$$\mathbf{q}^+(f_j)=[0 \quad 0 \quad \Pi^+(f_j) \quad -1]$$

We obtain the relationship :

$$\mathbf{q}^+(f_j)\ \mathbf{A}(f_j)=\Pi^+(f_j)\mathbf{A}_3(f_j)-\mathbf{A}_4(f_j)=0$$

The vector **q**(fj) is orthogonal to the steering vectors{**ap**(fj)}(p=1,.. ,P). In fact, the vector **q**(fj) belongs to the noise subspace as the smallest eigenvectors of the cross-spectral matrix.

To localise the sources the high resolution method is used. From the cleaned or unaffected block matrices of the spectral matrix the propagation operator is constructed .

Then, the orthogonality between **q**(fj) and A(fj) is exploited to localise the narrow-band or wide band sources impinging at the frequency fj, by plotting the function :

$$F(\theta)=\frac{1}{\left|\mathbf{q}^+(f_j)\,\mathbf{a}(f_j,\theta)\right|^2}$$

Note that with Music method[1] the function is :

$$F(\theta)=\frac{1}{\left|\mathbf{a}^+(f_j)\mathbf{V}_n(f_j)\mathbf{V}_n^+(f_j)\,\mathbf{a}(f_j,\theta)\right|}$$

where Vn(fj) is the noise subspace span by the N-P smallest eigenvectors of the spectral matrix.

4. OPERATOR ESTIMATION FROM DATA

In practice, the transfer matrix is unknown ; instead of the partition of A(fj) the partition of the received signal vector r(fj) is used to estimate the vector **q**(fj) or Π(fj). Let the four signal vectors $\mathbf{r}_1$(fj), $\mathbf{r}_2$(fj), $\mathbf{r}_3$(fj) and $\mathbf{r}_4$(fj) with P, K, P and 1 dimension respectively. Then, the block matrices of the cross-spectral matrix used for estimating the propagation operator are given by :

$$\Gamma_{13}(f_j)=\frac{1}{T}\sum_{t=1}^{T}\mathbf{r}_{1t}(f_j)\mathbf{r}_{3t}^+(f_j)$$

$$\Gamma_{14}(f_j)=\frac{1}{T}\sum_{t=1}^{T}\mathbf{r}_{1t}(f_j)\mathbf{r}_{4t}^+(f_j)$$

5. NUMERICAL EXAMPLE

We consider a linear array of N=14 equispaced sensors with equal interelement spacing d = c/2f_j. The source signals are temporally stationary zero-mean narrow-band. Four sources impinge on the array θ_1 =14°, θ_2 = 16°, θ_3 = 18° and θ_4 = 20° respectively, in the presence of the colored noise. The array noise is stationary zero-mean, independent of the signals, with penta-diagonal cross-spectral matrix with :

$$\Gamma_n(i,m)=\sigma^2\,\rho^{|i-m|}\,e^{j\pi(i-m)/2}$$

$$and\,\Gamma_n(i,m)=0 \quad if\ |i-m|>L$$

with the correlation length L = 5, the spatially correlation coefficient ρ = 1 and σ^2 =1 such that the signal to noise ratio is 10 dB.

The obtained results are shown figures 1 and 2. One can remark the Music method can not separate the four sources even the number of the sources is taken equal to 4, but the proposed method give the exact azimuth of the sources.

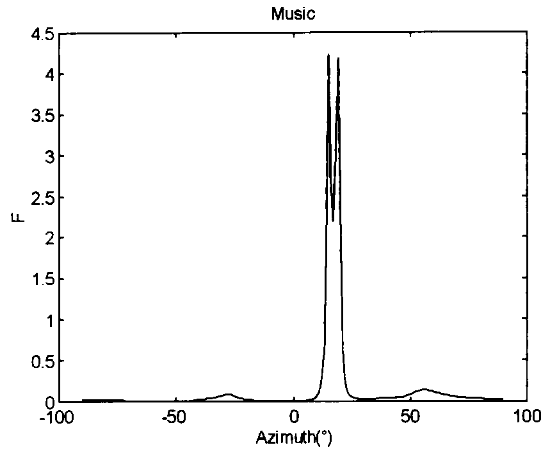

Figure1 : Music method with P= 4

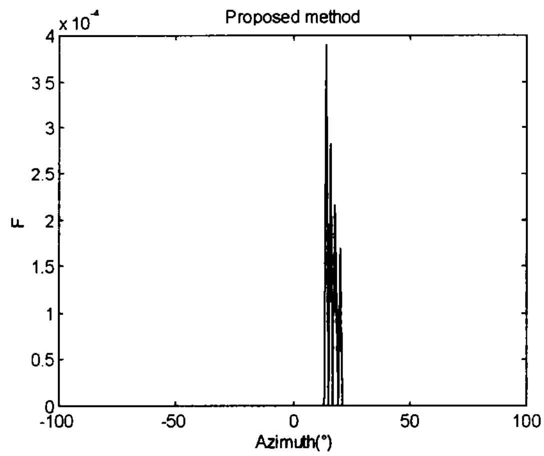

Figure 2 : Proposed method with K= L = 5

6. CONCLUSION

In this study, we have developed a new method for localisation of the narrow-band or the wide-band sources from the received data in the presence of unknown noise fields. An algorithm for estimating the noise subspace projection matrix without eigendecomposition is proposed. A linear operator estimated from the data is used instead of the eigenvectors ; this reduces considerably the computational load and permits us to implement it in real time. The simulation results show the new algorithm has , asymptotically, a performance similar to(i.e., exact and unbiased)the standard eigenstructure algorithm applicable in known noise fields.

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