ROBUST PEFORMANCE OF THE ADAPTIVE PERIODIC NOISE CANCELLER AS APPLIED TO ECHO CONTROL

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Abstract
The LMS-driven Adaptive Periodic Noise Canceller (APNC) can be used for acoustic echo suppression in the hands-free situation. Only recently has it been determined that the LMS algorithm can be robust [1]. The purpose of this contribution is to apply the concept of robustness, as derived from [1], to the APNC, thereby introducing a useful quantitative measure of performance.
This is achieved through an analysis of the performance of the APNC in an open-loop feedback system. Through the application of H^{∞} theory, conditions are shown under which the APNC, driven by the LMS algorithm, will exhibit robust performance properties. This has a direct application to the use of the APNC in echo control applications.

Keywords
Adaptive periodic noise canceller, echo cancellation, robust performance

1. Introduction

Perhaps the greatest technical challenge in the design of a hands-free speech terminal lies in the echo suppression device which must attenuate the electro-acoustic feedback of the far end speech signal. Robust performance of the Acoustic Echo Cancellation (AEC) device is essential in the context of hands-free telephony as the equipment must provide sufficiently high speech quality while dealing with sudden input disturbances [2].
Since the development of the LMS algorithm by Widrow [3], much research has investigated the conditions under which the algorithm will remain stable [4], [5], [6]. Unfortunately, algorithm stability will not guarantee robust performance. Work has also focused on the development of alternative echo cancellation algorithms and device configurations. Although some may demonstrate improved performance, a primary disadvantage with most methods is the increased complexity of implementation and sensitivity to changes at the input. This renders the system unsuitable in an environment where the feedback echo can exhibit rapid fluctuations of magnitude.
It has been long known, especially experimentally, that LMS driven adaptive filter systems perform best in the most adverse conditions. In relation to the input signal-to-noise ratio (SNR), this observation could be justified theoretically. However, the experimentally observed robustness of such LMS adaptive systems has been known to extend beyond simple SNR considerations.
The robust performance analysis of the LMS algorithm provided in [1] initiated the process of establishing the formal link between H^{∞} theory and the experimentally observed robustness of the LMS algorithm. This contribution intends to apply the ideas developed to the specific case of the APNC embedded in an open-loop system. The intention is to model the practical situation where the APNC is applied to cancel the echo component at its input.

2. Theoretical Development

The APNC-based echo control system is modelled as in Fig.1 below. $S(n)$ denotes the speech input and the echo is represented by the output $u(n)$ of an auto-regressive (AR) process, where the feedback path in turn represents the acoustic environment. The analysis below assumes for simplicity that there is no speech signal present at the APNC input.

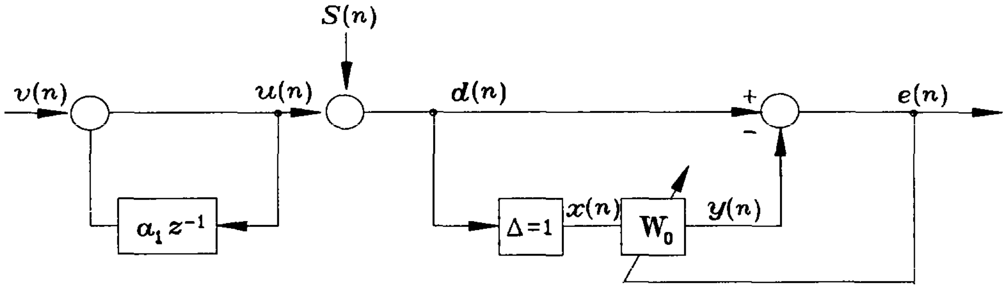

Fig.1 Block Diagram of Simple APNC and Echo Model

The APNC output $e(n)$ is given by

$$e(n) = u(n) - W_{0n}u(n-1) \quad (1)$$

where W_{0n} is the APNC weight, and $u(n)$ is formed from

$$u(n) = v(n) - a_1 u(n-1) \quad (2)$$

where $v(n)$ is Gaussian white noise and a_1 is the AR(1) coefficient. With $x(n) = u(n-1)$, (1) may be rewritten as

$$e(n) = v(n) - (a_1 + W_{0n})x(n) \quad (3)$$

In the ideal situation, the weight value cancels with the AR(1) coefficient leaving only the generating white noise at the output [7]. The second term in (3) gives the residual output error as a function of the filter input. If the system is to exhibit robust performance, the energy of this error should be minimised by the adaptive solution. The LMS solution is dependent on the choice of stepsize μ and for robust performance it is necessary to determine the maximum upper limit for μ.

Denoting the error term as $\varepsilon(n)$ and $B_n = -(a_1 + W_{0n})$, the error at time n is

$$\varepsilon(n) = B_n x(n) \quad (4)$$

B_n can be interpreted as a time-varying 'output error gain' and it is a function of all previous inputs and outputs. Robust performance would imply that the magnitude of this gain is minimised over time and that signal fluctuations at the input will not disturb this minimisation procedure.

The LMS algorithm is given by

$$W_{0n} = W_{0n-1} + 2\mu e(n-1)x(n-1) \quad (5)$$

Adding a_1 to both sides and negating, (5) can be expressed in terms of B_n,

$$B_n = B_{n-1} - 2\mu e(n-1)x(n-1) \quad (6)$$

Then, substituting from (3),

$$B_n = B_{n-1}\left(1 - 2\mu x^2(n-1)\right) - 2\mu v(n-1)x(n-1) \quad (7)$$

The evolution of B_n is given by (7). Ideally, $\lim_{n\to\infty} B_n = 0$ but in reality there will always be a small misadjustment, or estimation error, which is dependent on the terms $v(n-1)$ and $x(n-1)$. For robust performance, the energy of the misadjustment must not be greater than the energy of the components which created it. To examine how the misadjustment is sensitive to changes in the magnitude of these terms, the transfer function between the input and output must be described in matrix form in order to account for the system memory, i.e. show the evolution of B_n over time.

(4) can be represented as a time-varying matrix

$$\boldsymbol{\varepsilon}(n) = \boldsymbol{B}_n \boldsymbol{X}(n) \quad (8)$$

where $\boldsymbol{\varepsilon}(n)$ denotes a vector of error outputs to time n.

By finding the H^∞-norm of $\boldsymbol{B}$, the peak value of the gain over the time interval $n = 1, \ldots, N$, can be determined. This is defined as

$$\|B\|_\infty = \sup_{x\neq 0} \frac{\|\varepsilon\|_2}{\mu^{-1}|B_1| + \|X\|_2} \qquad (9)$$

where the input disturbance is $\left\{\mu^{-1/2}(B_1), \{x(n)\}_{n=1}^{\infty}\right\}$ with $\mu^{-1/2}(B_1)$ being the (weighted) energy of the weight error due to the initial guess and the h_2 - norm of x_k is given as $\|X\|_2^2 = \sum_{k=1}^{\infty} x_k^* x_k$.

For robust performance the energy of the residual error should be upper bounded by the energy of the disturbances and the initial uncertainty. This translates into ensuring that the H^∞-norm of the residual error gain matrix (**B**) must be less then one,

$$\|\boldsymbol{B}(\mu)\|_\infty \leq 1 \qquad (10)$$

where $\boldsymbol{B}(\mu)$ indicates the dependence of **B** on the stepsize μ .

The H^∞-norm is calculated by finding the maximum singular values $\sigma_{n\max}$ of **B** at each time instant and are given by

$$\sigma_{n\max} = \sqrt{\max(\lambda_n(\boldsymbol{B}\boldsymbol{B}^T))} \qquad (11)$$

The matrix description of the system is given below

$$\begin{bmatrix} \varepsilon(1) \\ \varepsilon(2) \\ \vdots \\ \varepsilon(N) \end{bmatrix} =$$

$$\begin{bmatrix} \mu^{1/2}x(1) & 0 & \cdots & 0 \\ \mu^{1/2}x(2) & -2\mu v(1)x(2) & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ \mu^{1/2}x(n)\prod_{k=1}^{N-1}(1-2\mu x^2(k)) & -2\mu v(1)x(n)\prod_{k=2}^{N-1}(1-2\mu x^2(k)) & \ldots & -2\mu v(n-1)x(n) \end{bmatrix}$$

$$\times \begin{bmatrix} \mu^{-1/2}B_1 \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} \qquad (12)$$

From (12), it is obvious that stepsize choice is a crucial factor affecting the maximum singular values. Therefore, substituting (7) into (4) and focusing on the output error gain, this leads to

$$\left|B_{n-1} - 2\mu\left(x^2(n-1)B_{n-1} + x(n-1)v(n-1)\right)\right| \leq 1 \qquad (13)$$

To ensure that the energy of the disturbance $x(n)$ is not amplified, the stepsize must be chosen so that

$$\left|-2\mu\, v(n-1)\, x(n)x(n-1)\right| \leq 1 \qquad (14a)$$

and

$$\left|-2\mu\, B_{n-1}x^2(n-1)\right|^2 \leq 1 \qquad (14b)$$

Empirically, the following choices of upper bound for the algorithm stepsize parameter were made to examine if robust behaviour was ensured,

$$\mu \leq 1 \Big/ \left(2 \times v_{\max} \times x_{\max}^2\right) \tag{15a}$$

and,

$$\mu \leq \left(1-a_1\right) \Big/ \left(2 \times x_{\max}^2\right) \tag{15b}$$

3. Results

Simulations were carried out for the system shown in Fig.1 to test its robust performance qualities. The APNC input was an 400-point AR(1) process and with each simulation the value of the AR(1) coefficient was varied over the range 0.2 to 0.9. The maximum singular values ($\sigma_{n\max}$) of $B(\mu)$ were found for each iteration to check if (10) was satisfied. The stepsizes chosen were as given in (15a) and (15b) above. For the choice of stepsize (15b), condition (10) is indeed satisfied, as shown in Fig.3. For the choice (15a) it can be seen in Fig.2 that the $\sigma_{n\max}$ are greater than unity for the smaller values of AR(1) coefficient. From Fig.3, it can be observed that the stepsize value (15b) gives faster algorithm convergence and a better reduction in the output error $\varepsilon(n)$. Therefore, it can be concluded that Stepsize (15b) demonstrates better robustness performance properties than (15a). Overall, both graphs show that the APNC performs best under the worst case conditions.

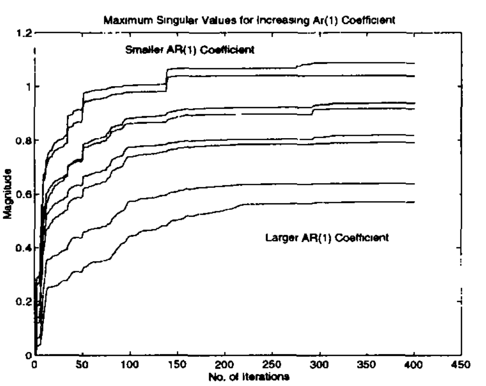

Fig.2 $\sigma_{n\max}$ of B with Stepsize (15a)

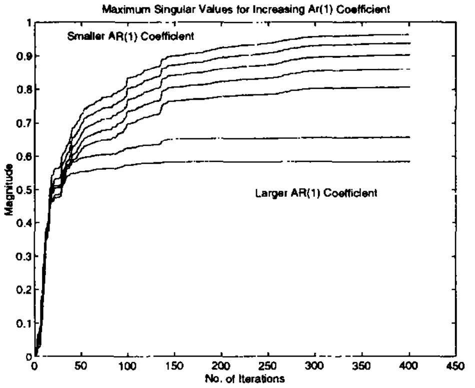

Fig.3 $\sigma_{n\max}$ of B with Stepsize (15b)

4. Conclusion

In general, this investigation has provided an alternative method under which the echo cancelling performance of the LMS algorithm can be analysed. The criteria for robustness has been shown to be a more stringent condition for algorithm assessment and, more importantly, a practically meaningful quantity.

5. References

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