PERFORMANCE ASSESSMENTS OF FIR VERSUS IIR MODELS IN ACOUSTIC ECHO CANCELLATION

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ABSTRACT

The adequateness of IIR models for acoustic echo cancellation is a long standing question and the answers found in the literature are conflicting. We use results from rational Hankel norm and least-squares approximation and we recall a test which provides *a priori* performance levels for FIR and IIR models. We apply this test to measured acoustic impulse responses. Upon comparing the performance levels of equal complexity FIR and IIR models, we do *not* observe any significant gain from the use of IIR models. We attribute this phenomenon to the shape of the energy spectra of the acoustic impulse responses, so tested, which possess many strong and sharp peaks. Faithful modelling of these peaks requires many parameters irrespective of the type of the model.

1. INTRODUCTION

The use of adaptive FIR filters for Acoustic Echo Cancellation leads to filters with very high orders and consequently the adaptive adjustment of their coefficients leads to very high computational complexity [1]. There exist several works in the literature which, based partially on intuition and partially on theoretical results, claim the adaptive IIR filters to outperform equal complexity adaptive FIR filters [2]. However, succesfull application of adaptive IIR algorithms for acoustic echo cancellation has not been achieved [3], [4]. There exist many reasons which may be at the root of this failure:

- It is not trivial to guarantee that an adaptive IIR algorithm will approach the best possible performance an IIR model can offer, because of possible existence of *local minima*.
- It is not trivial to guarantee *stability* of the adaptive IIR filter during the adaptation process.
- The *convergence speed* of adaptive IIR algorithms may be lower than that of their FIR counterparts.
- The IIR models *do not* offer better modelling capabilities than their FIR counterparts; if this is the case, isolate the causes of this phenomenon, giving an end to the discussion about the *adequateness* of IIR models for acoustic echo cancellation.

In order to give a complete answer to the question of the succesfull application of adaptive IIR algorithms to the problem of acoustic echo cancellation, we must study all the aforementioned subproblems.

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2. LEAST-SQUARES APPROXIMATION USING FIR AND IIR MODELS

We start by considering the last subproblem, which appears to be the most important. Let us consider the system identification setup shown in Figure 1. The unit variance zero mean white noise sequence $u(n)$ drives both $H(z)$ and $\widehat{H}(z)$. We assume that $H(z)$ is a causal and stable (in the l_2 sense) system:

$$H(z) = \sum_{k=0}^{\infty} h_k z^k, \quad \text{with} \quad \sum_{k=0}^{\infty} h_k^2 < \infty, \qquad (1)$$

where $z^k u(n) = u(n-k)$. Its output $y(n)$ can be expressed in operator notation as

$$y(n) = H(z)u(n) = \sum_{k=0}^{\infty} h_k u(n-k). \qquad (2)$$

$\widehat{H}(z)$, our adjustable model, is constrained to be causal. It may be either an M-th order FIR filter

$$\widehat{H}(z) = \sum_{k=0}^{M} \hat{h}_k z^k, \qquad (3)$$

or a K-th order IIR filter

$$\widehat{H}(z) = \frac{B(z)}{A(z)} = \frac{\sum_{k=0}^{K} b_k z^k}{\sum_{k=0}^{K} a_k z^k} = \sum_{k=0}^{\infty} \hat{h}_k z^k. \qquad (4)$$

Its output $\hat{y}(n)$ is used as an estimate of $y(n)$.

Our objective is to determine the filters $\widehat{H}(z)$ which minimize the mean square estimation error,

$$\min_{\widehat{H}(z)\in\mathcal{H}} E\left[e^2(n)\right], \tag{5}$$

where $\mathcal{H}$ denotes the class of the models we use (either FIR or IIR). Since our input is unit variance zero mean white noise, this minimization problem reduces to

$$\min_{\widehat{H}(z)\in\mathcal{H}} \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega})-\widehat{H}(e^{j\omega})\right|^2 d\omega \tag{6}$$

or equivalently

$$\min_{\widehat{H}(z)\in\mathcal{H}} \sum_{k=0}^{\infty}\left(h_k-\hat{h}_k\right)^2. \tag{7}$$

2.1 M-th order FIR case

When $\widehat{H}(z)$ is an M-th order FIR model, our minimization problem becomes

$$\min_{\hat{h}_k,\ k=0,\ldots,M}\left(\sum_{k=0}^{M}(h_k-\hat{h}_k)^2+\sum_{k=M+1}^{\infty}h_k^2\right). \tag{8}$$

It is clear that the coefficients of the optimum M-th order FIR filter match the first $M+1$ coefficients of $H(z)$, giving

$$\min_{\hat{h}_k,\ k=0,\ldots,M} E\left[e^2(n)\right]=\sum_{k=M+1}^{\infty}h_k^2. \tag{9}$$

Thus, if we assume that we know the impulse response h_k, $k=0,1,\ldots$, we can compute *a priori* the best performance achieved by FIR models, as a function of the model order.

2.2 K-th order IIR case

When $\widehat{H}(z)$ is a K-th order IIR model, the minimization problem (7) becomes

$$\min_{\deg\widehat{H}(z)\leq K} \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|H(e^{j\omega})-\widehat{H}(e^{j\omega})\right|^2 d\omega. \tag{10}$$

The performance level achieved by the optimum K-order IIR filter is not given as simply as in the FIR case. Actually, we cannot derive, in general, exact expressions for the minimum mean squared error in terms of the impulse response of $H(z)$. What we can obtain, assuming we know h_k, $k=0,1,\ldots$, is a priori upper and lower bounds for the minimum mean square error. These bounds depend on the Hankel singular values of $H(z)$, which are defined as follows. Given a stable and causal $H(z)$ as in (1), its Hankel form is defined as the doubly infinite Hankel matrix

$$\Gamma_H=\begin{bmatrix} h_1 & h_2 & h_3 & \cdots \\ h_2 & h_3 & h_4 & \cdots \\ h_3 & h_4 & h_5 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{bmatrix}. \tag{11}$$

The Hankel singular values of $H(z)$ are the singular values of Γ_H, $\sigma_i(\Gamma_H)$, and they are usually given in descending order.

It may be shown that in the simple case of zero mean unit variance white noise input [5, pp. 168]

$$\min_{\deg\widehat{H}(z)\leq K} E\left[e^2(n)\right]\leq\sigma_{K+1}(\Gamma_H). \tag{12}$$

It may also be shown [6] that

$$\min_{\deg\widehat{H}(z)\leq K} E\left[e^2(n)\right]\geq\left(\sum_{i=K+1}^{\infty}\sigma_i^2(D\Gamma_H D)\right)^{1/2}, \tag{13}$$

where $D=\mathrm{diag}(d_0,d_1,\ldots)$, with

$$d_k=d_{k-1}\sqrt{\frac{2k-1}{2k}},\quad d_0=1. \tag{14}$$

Thus, assuming that we know h_k, $k=0,1,\ldots$, we can derive *a priori* upper and lower bounds for the performance levels offered by the IIR models, as a function of the model order K.

These bounds refer to the global minimum of the error performance surface, that is, they refer to the best possible performance IIR models can offer, and are independent of the existence of local minima.

In the sequel we use (9), (12) and (13) in order to compare the modelling capabilities of IIR versus FIR models for acoustic echo cancellation.

In Figure 2 we plot the samples of the impulse response of the acoustic echo path (sampling frequency 8 KHz); in Figure 3 we plot the magnitude of its energy spectrum in the frequency range 100 to 1000 Hz.

In Figure 4 we plot the performance levels offered by the models, as a function of the number of the model parameters. The thick lines plot the upper and lower least-squares error bounds for IIR models, while the thin line plots the minimum least-squares error achieved by the respective FIR models. We observe that for parameters number up to 1500, the IIR models cannot claim to offer substantially improved modelling capabilites than their equal complexity FIR counterparts.

We applied the same test to numerous acoustic impulse responses [1]. We always observed the same phenomenon; that is, in none of the cases the IIR models did *not* offer much better modelling capabilities than equal complexity FIR models.

3. ADEQUATENESS OF IIR MODELS FOR ACOUSTIC ECHO CANCELLATION

In the previous section we observed that IIR models do not offer better modelling capabilities than their FIR counterparts, in the acoustic echo cancellation context. It seems very interesting to isolate those characteristics of acoustic echo paths which seem to be the main causes for this phenomenon. This is our objective in this section.

Staring at the impulse response plotted in Figure 2, we observe a *decreasing exponential envelope*, which has justified the use of IIR models for the modelling of acoustic echo paths [2].

Staring at the magnitude of the energy spectrum of this impulse response, in Figure 3, the most striking observation is the existence of *many strong sharp* spectral peaks. As a result, for this particular energy spectrum, there exist more than 1000 extrema points in the frequency range 0 to 4000 Hz. In the sequel we show, that in order to model this energy spectrum "perfectly" we need *at least* 1000 parameters, irrespective of the type of the model used.

Consider first the FIR case. In order to compute the maximum number of extrema of

$$\left|\widehat{H}(e^{j\omega})\right|^2 = \widehat{H}(e^{j\omega})\widehat{H}(e^{-j\omega}), \qquad (15)$$

on the interval $[0,\ \pi]$, with

$$\widehat{H}(z) = \sum_{k=0}^{M} \hat{h}_k z^k, \qquad (16)$$

we first write

$$\left|\widehat{H}(e^{j\omega})\right|^2 = \sum_{k=0}^{M} \alpha_k \cos(\omega k), \qquad (17)$$

for some α_k, $k = 0, \ldots, M$. Then, we follow the same steps as in [7, p. 128] and we conclude that the maximum number of extrema of $\left|\widehat{H}(e^{j\omega})\right|^2$ on the interval $[0,\ \pi]$ is $M + 1$. Using similar arguments we can prove that the maximum number of extrema points of $\left|\widehat{H}(e^{j\omega})\right|^2$, where $\widehat{H}(z)$ is the K-th order IIR model given by (4), is $2K + 1$.

This means that the minimum number of parameters required for modelling "perfectly" an energy spectrum, whose magnitude possesses M extrema points on the interval $[0,\ \pi]$, is equal to $M - 1$, *irrespective* of the type of the model.

The shape of the magnitude of the energy spectrum of the acoustic echo path, that is the existence of *many strong and sharp* peaks, implies that in order to provide *good* least-squares approximations we must model, somehow, *many* spectral peaks. From the previous discussion it is obvious that the modelling of many peaks requires many parameters – because we must provide many extrema points – irrespective of the type of the model. Thus, in order to provide *good acoustic echo path approximations*, we must use an IIR model with a *very large number of parameters.*

We may make an impression for the approximation properties of the models in the acoustic echo cancellation problem by looking at Figure 6. With the solid line we plot the magnitude of the energy spectrum of the acoustic impulse response, while with the thick and dotted thick lines, respectively, we plot the optimum FIR and IIR approximations; the optimum IIR approximation has been derived by using a combination [8] of the Steiglitz-McBride and the partial gradient IIR algorithm [5, Ch. 7, 8]; the number of parameters is 400 for both models and the frequency range is from 600 to 1000 Hz. We observe that both models provide *smooth* approximations. In general the energy spectrum of the optimum FIR possesses more extrema points than that of the optimum IIR model.

4. CONCLUSIONS

Using theoretical results from the rational Hankel and least-squares aproximation theories, we recalled a test which can be used to derive a priori performance levels for these models, as a function of the number of the model parameters. Applying this test to a *number* of measured acoustic impulse reponses, we did *not* observe any substantial improvement by the use of IIR models.

The main reason for this fact lies, in our opinion, in the shape of the energy spectra of the acoustic echo

[1] These acoustic impulse responses are furnished courtesy of Dr. E. Hansler, and Dr. R. Martin.

paths, so tested. Their striking characteristic is the existence of many strong sharp spectral peaks. The modelling of these peaks requires many parameters, irrespective of the type of the model [8].

Thus, regrettably, it seems unlikely that we will manage to develop techniques based on IIR models which will outperform significantly the corresponding techniques based on equal complexity FIR models, in the acoustic echo cancellation problem.

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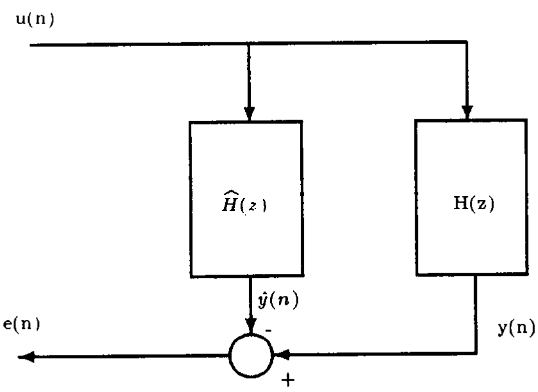

Fig. 1 System identification framework.

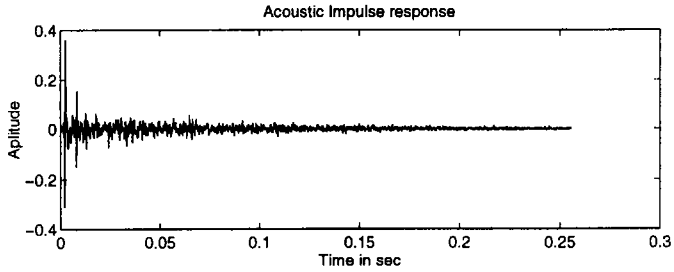

Fig. 2 Acoustic impulse response.

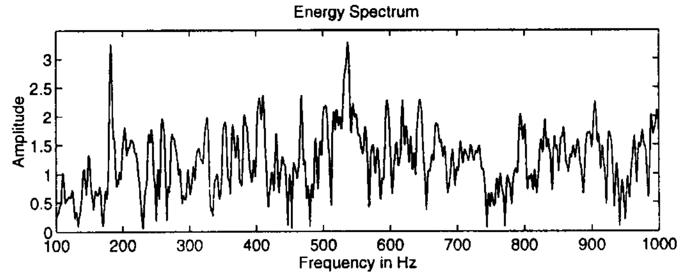

Fig. 3 Energy spectrum of the acoustic impulse response of Fig. 1 (100-1000Hz).

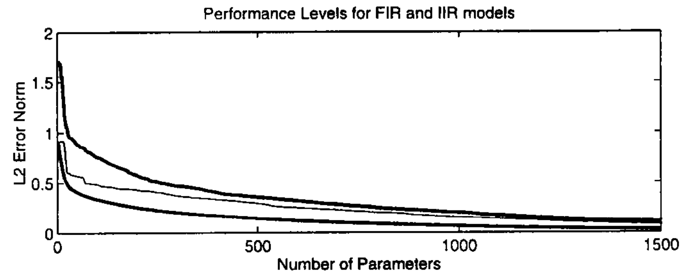

Fig. 4 Dark lines: upper and lower bounds on attainable approximation error for the IIR case; Middle line: exact bound for FIR case.

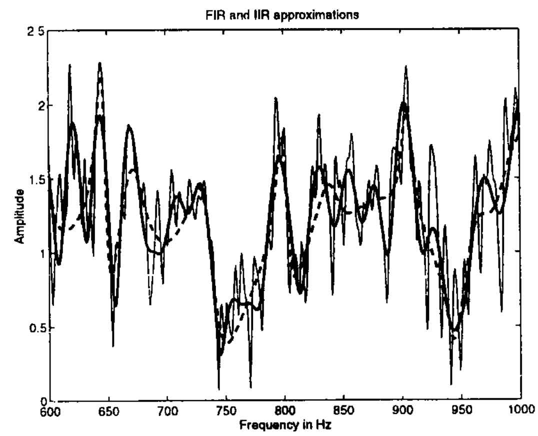

Fig. 5 Solid line: Energy spectrum of acoustic impulse response. Thick line: FIR approximation. Dotted thick line: IIR approximation.